وزارة التعليم العالي والبحث العلمي

الجامعة التقنية الجنوبية

المعهد التقني/الشطرة

قسم الكهرباء/الشبكات

Electrical Circuits

الدوائر الكهربائية

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Chapter One

Basic Concept and Units

1-1-The SI System of Units

This lectures employs the International System of Units, commonly called SI (standard international) .SI units are based on six fundamental quantities, listed in Table 1.1.

Standard prefixes are used to denote powers of 10 of SI (and derived) units. These prefixes are listed in Table 1.2. The units in Table 1.3 are derived units.

Table 1.1 SI units

Quantity	Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Luminous intensity	Candela	cd

Table 1.2 Standard prefixes

Prefix	Symbol	Power
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

Common Power of Ten Multipliers

$1\ 000\ 000 = 10^{6}$	$0.000001 = 10^{-6}$
$100\ 000 = 10^{5}$	$0.00001 = 10^{-5}$
$10\ 000 = 10^{4}$	$0.0001 = 10^{-4}$
$1\ 000 = 10^{3}$	$0.001 = 10^{-3}$
$100 = 10^{2}$	$0.01 = 10^{-2}$
$10 = 10^{1}$	$0.1 = 10^{-1}$
$1 = 10^{0}$	$1 = 10^{0}$

Similarly, the number 0.003 69 may be expressed as 3.69×10^{-3} as illustrated below.

$$0.003\ 69 = 0.\underbrace{0\ 0\ 3}_{1\ 2\ 3}\ 6\ 9 = 3.69 \times 10^{-3}$$

TABLE 1–3 Some SI Derived Units

Quantity	Symbol	Unit	Abbreviation
Force	F	newton	N
Energy	W	joule	J
Power	P, p	watt	W
Voltage	V, v, E, e	volt	V
Charge	Q, q	coulomb	C
Resistance	R	ohm	Ω
conductance	G	mho	℧
Capacitance	C	farad	F
Inductance	L	henry	H
Frequency	f	hertz	Hz
Magnetic flux	Φ	weber	Wb
Magnetic flux density	B	tesla	T
Efficiency	η		
Kilo watt hour	Kwh		
Horse power	hP		

1kwh = 3.6×10^6 J

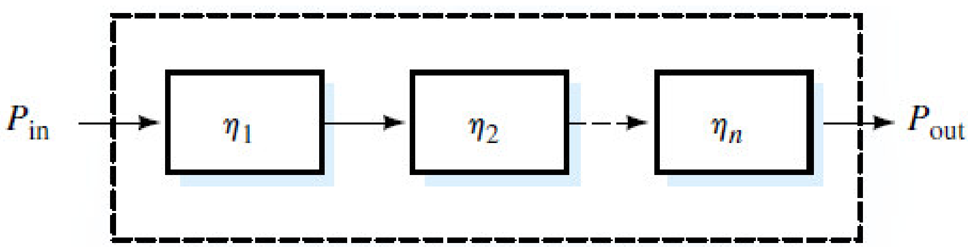
= 3.6 M J.

1 joule = 1 N-m.

1hp = 746 watts.

1kwh =1000 wh. =1000×60 w.min =1000×60 × 60 w.sec =3.6×10^6 J.=3.6 M.J.

$\eta = p_o / p_{in}$



$\eta_T = \eta_1 \times \eta_2 \times \ldots \times \eta_n$, $\eta_T = p_o / p_{in}$

$w = F\ell = P\,t = Q\,V$

$P = V\,I$, $I = Q/t$

Example: what are the value of the work done in jule to move one electron through a potential difference 100 v.

solution:

$$w = F\ell = P\,t = V\,I\,t = (Q/t)\,V\,t = QV = 1.6 \times 10^{-19} \times 100 = 1.6 \times 10^{-17}$$

1-2 Basic Concepts

1- Voltage (or potential difference) is the energy required to move a unit charge of one coulomb from one point to another through an element, the potential difference between the points is one volt, measured in volts (V).

1V = 1 J/C

2- Electric current is the time rate of change of charge or rate of flow of electrical charges in a circuit, measured in amperes (A).

1A= 1 C/S

3- Current Density is the intensity current the pass by unit of the vertical area on the direction current .

J= I / A

4- Power and Energy
Power is the time rate of expending or absorbing energy, measured in watts (W).It is also the product of voltage and current P= VI .
Energy is the capacity to do work, measured in joules (J).

5- Resistance is defined as the opposition to current flow or denotes its ability to resist the flow of electric current,we find that the resistance of a material is dependent upon several factors:
• Type of material
• Length of the conductor
• Cross-sectional area
• Temperature

R=ρ ℓ / A

6- conductance can be defined as the ability to conduct current , or the reciprocal of resistance. It is measured in mhos (℧) or siemens (S).
G=1 / R

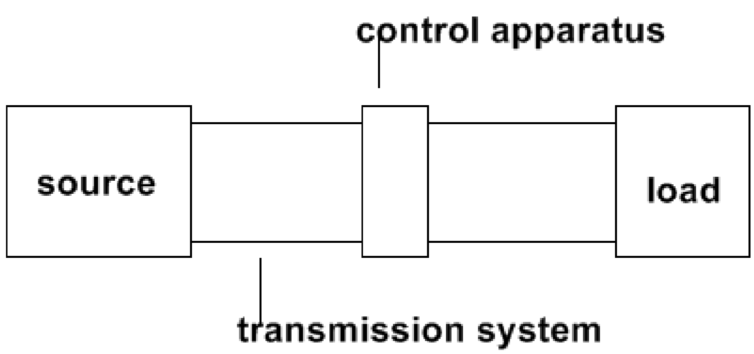
•Conductivity and Resistivity
σ : is known as the **conductivity**, where is factor of the conductance .It is measured in mhos per meter
ρ : is the constant and known as the **resistivity** or specific resistance of the material in ohm-meters.
Table 1.4 presents the values of ρ for some common materials and shows which materials are used for conductors, insulators, and semiconductors.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.83×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{-2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

TABLE 1.4 Resistivities of common materials.

1-3 Electric Circuit

An electric circuit consists of electrical elements connected together in any manner which allows charge to flow(closed path),and electrical elements this sources , loads,transmission system and control apparatus.As in the figure shown below.

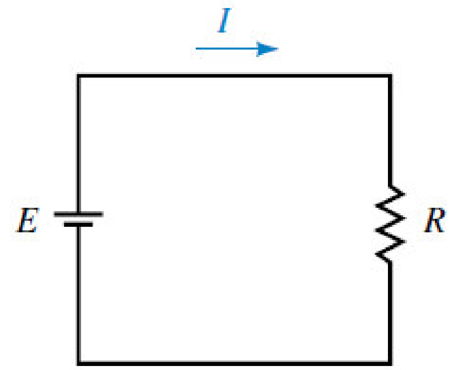


source : are voltage or current sources that generally deliver power to the circuit connected to them.
load : are resistors or capacitors or inductors or connected together in any manner .
transmission system : it is to connect the electrical elements together , there are often on form wires.
control apparatus : to control on the electrical current flow in the circuit .

1- 4 Ohm's Law

states that the voltage *v* across a resistor is directly proportional to the current *i* flowing through the resistor.

That is, $V \alpha I$



Circuit for illustrating Ohm's law.

Ohm defined the constant of proportionality for a resistor to be the resistance,R.

Thus, Eq. becomes

V=I R [volts,V]

Ohm's law states

I = V/ R [amps,A]

And

R=V/I [ohms,Ω]

1-5 Power

It is also the product of voltage and current .

$P= VI$, $P= I^2 R$ and $P=V^2 / R$

1-6 Base Electrical Units Definition

1- Ampere (A or amp):-

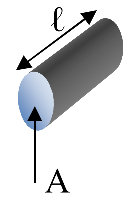
The SI unit of electrical current, equal to a rate of flow of one coulomb of charge per second.

2- volt (V):-
The unit of voltage in the SI system, the voltage between two points on the conducter is one volt .
3- coulomb (C) :-
The SI unit of electrical charge, is defined as the charge carried by constant current (1amp) in second , the charge on one electron. It is 1/(6.24 ×1018) = 1.60×10^{-19} C.

4- ohm (Ω) :-
The SI unit of resistance. Also used as the unit for reactance and impedance ,equal to one volte over current (1amp) , 1Ω =1V/I

Examples

1- Most homes use solid copper wire having a diameter of 1.63 mm to provide electrical distribution to outlets and light sockets. Determine the resistance of 75me ters of a solid copper wire having the above diameter.



Solution : The resistance is determined as

$R=\rho\ell/A$

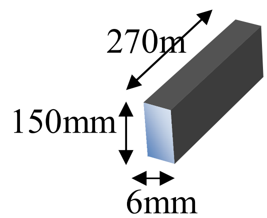
The cross-sectional area is

$A= \pi r^2 = \pi (d / 2) = \pi d^2 / 4 = \pi (1.63 \times 10^{-3} \text{ m})^2 / 4 = 2.09\times10^{-6} \text{ m}^2$

The resistance of the length of wire is found as

$R=\rho\ell/A = (1.723\times10^{-8}\ \Omega\text{-m})(75\text{m})/A = (1.723\times10^{-8}\ \Omega\text{-m}^2) / 2.09\times10^{-6} \text{ m}^2 = 0.619\Omega$

2-Bus bars are bare solid conductors (usually rectangular).Given a piece of aluminum bus bar as shown in Figure below, determine the resistance between the ends of this bar at a temperature of 20°C.



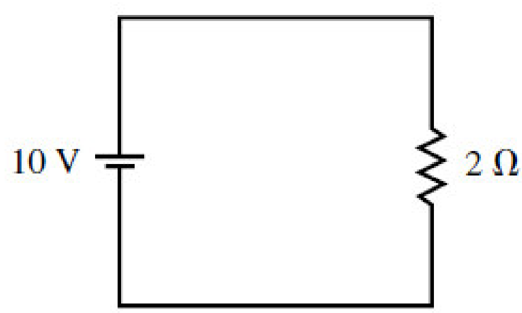
solution : The cross-sectional area is

$A = 150\text{mm} \times 6\text{mm} = 150\times10^{-3} \times 6\times10^{-3} \text{ m}^2$
$= 900\times10^{-6} \text{ m}^2 = 9\times10^{-4} \text{ m}^2$

$R=\rho\ell/A$
$=(2.825\times10^{-8}\ \Omega\text{-m})(270\text{m})/A$

$=(7.6275\times10^{-6}\ \Omega\text{-m}^2) / (9\times10^{-4} \text{ m}^2)$
$= 8.475\times10^{-3}\ \Omega = 8.48 \text{ m}\Omega$

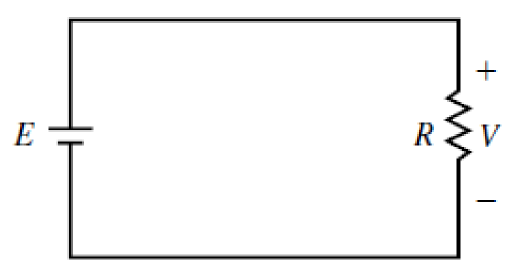
3- For the Figure shown below .What is the current?



Solution : By using Ohm's law

$I=V/R = 10 / 2 = 5\text{A}$

4-For the figure shown below a resistor draws 3 A from a 12-V battery. How much power does the battery deliver to the resistor ?



Solution : The power deliver to the resistor is
$P = V I= 12\times 3=36\text{W}$
$P= I^2 R = 9\times (12/3) =36\text{W}$

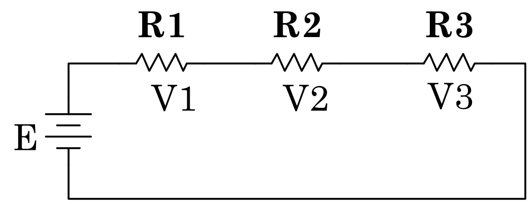
Chapter Two
Basic Laws of D.C. Circuit

2-1 Types of D.C. Circuits :

DC circuits can be classified as :

1- Series Circuits.
2- Parallel Circuits.
3- Series-Parallel Circuits.

1- Series Circuit



$R_T = R_1+R_2+R_3$

$E = V_1+V_2+V_3$

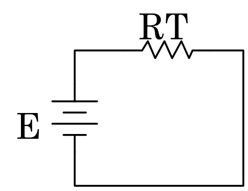
$= I_1 R_1+I_2 R_2+I_3 R_3$

$I_1=I_2=I_3= I$

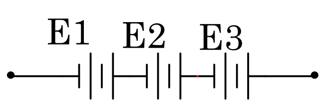
The voltage source, E,is

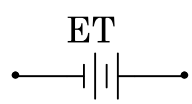
$E = I (R_1+R_2+R_3)$

$= I R_T$



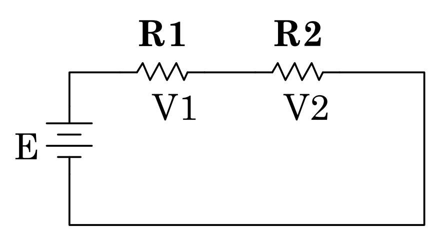
• Voltage Sources in Series





$E_T =E_1+E_2+E_3$

• Voltage Divider Rule



$I=V/(R_1+R_2)$

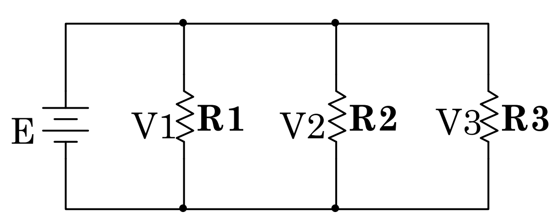
$V_1 = I R_1 = (V/(R_1+R_2)) R_1$

or $V_1=V R_1 /(R_1+R_2)$

Similarly

$V_2 = V R_2 / (R_1+R_2)$

2- Parallel Circuits



$1/R_T = 1/R_1 + 1/R_2 + 1/R_3$

$I_T = I_1+ I_2+ I_3$

$= V_1/ R_1 + V_2/ R_2 + V_3/ R_3$, $E=V_1=V_2=V_3 = V$

$= V (1/ R_1 + 1/ R_2 + 1/ R_3)$, $I = V / R_T$

By using the conductance (G)

$G = 1/R$

$G_T = G_1 + G_2 + G_3 = 1/R_T$

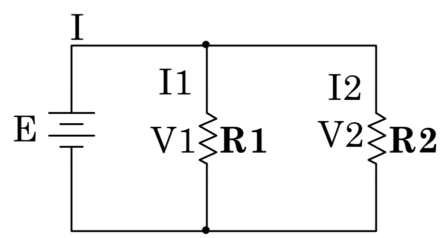
- For two resistors in parallel,then R_T given as :

$R_T = (R_1 \times R_2) / (R_1 + R_2)$

- For three resistors in parallel,then R_T given as :

$R_T = (R_1 \times R_2 \times R_3) / (R_1 R_2 + R_1 R_3 + R_2 R_3)$

- Current Divider Rule



$R_T = (R_1 R_2) / (R_1 + R_2)$

$R_T = R_{eq}$

$I_1 = V / R_1 = I R_{eq} / R_1$

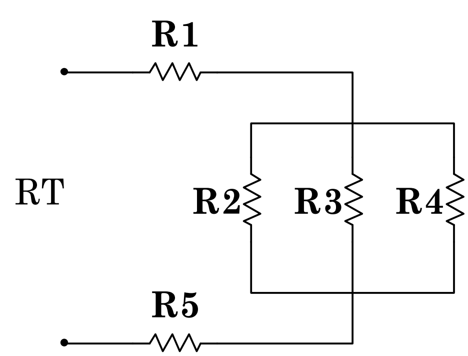
$= I R_2 / (R_1 + R_2)$

Similarly

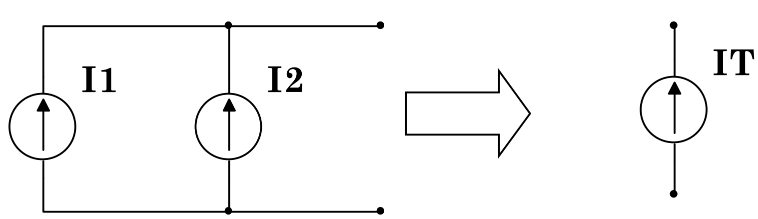
$I_2 = V / R_2 = I R_1 / (R_1 + R_2)$

3- Series-Parallel Circuits

$R_T = R_1 + (R_2 // R_3 // R_4) + R_5$



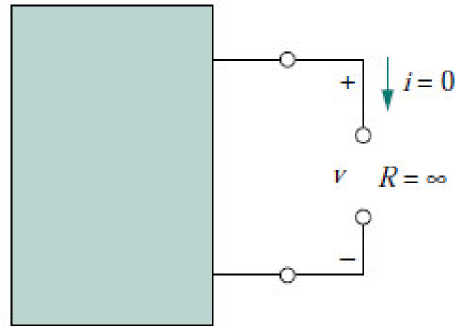
- Current Sources in Parallel



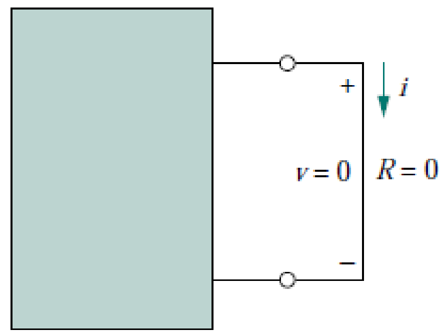
$I_T = I_1 + I_2$

2-2 Open and Short Circuits

A circuit element with resistance approaches infinity is called an **open circuit.**
The open circuit:
$R \rightarrow \infty$
I = 0 for any V

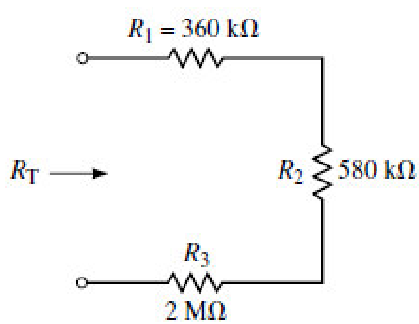


A circuit element with resistance approaching zero is called a **short circuit**.
The short circuit:
R = 0
V = 0 for any I

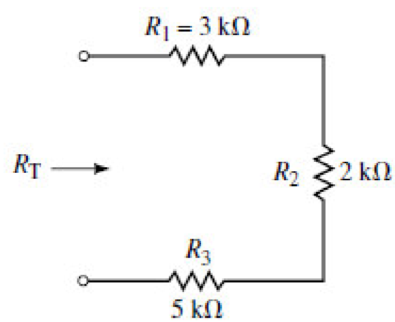


Example: Determine the total resistance of the networks shown in Figures below **.**

a-



b-



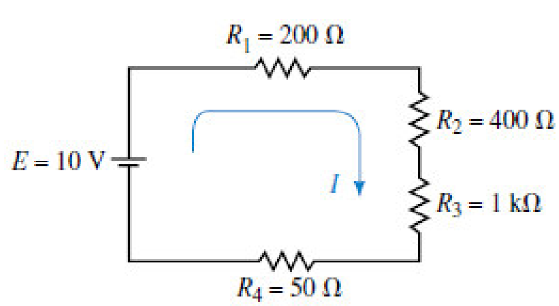
$R_T = R_1 + R_2 + R_3$
$= (360+580)10^3 + 2\times10^6$
$= 2.94\ M\Omega.$

$R_T = R_1 + R_2 + R_3$
$= (3+2+5)\times 10^3 = 10\ K\Omega.$

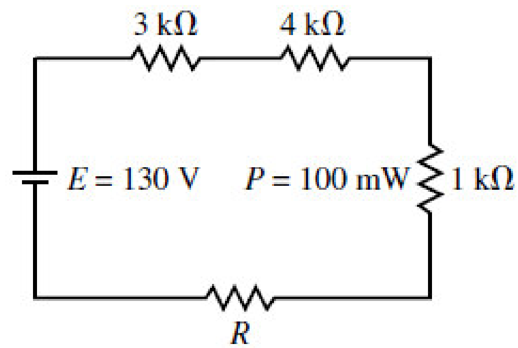
Example : For the circuit shown in Figure below, determine the total resistance, RT,and the current, I.

Solution : $R_T = 200+400 + 50 + 1000$
$= 1650\ \Omega = 1.65\ K\Omega.$

$I = E / R_T = 10 / 1650 = 6.06 \times 10^{-3}$ A
$= 6.06$ mA



Example : For the circuit of Figure, find the following quantities:
a. The circuit current.
b. The total resistance of the circuit.
c. The value of the unknown resistance, *R*.
d. The voltage drop across all resistors in the circuit.
e. The power dissipated by all resistors.



Solution :

a- $P_3 = 100\times10^{-3} = I^2 R$

$I = \sqrt{((100 \times 10^{-3})/1000)} = 10mA$

d- $V_1 = 0.01\times 3000 = 30V$

$V_2 = 0.01\times 4000 = 40V$

$V_3 = 0.01\times 1000 = 10V$, $V_4 = E- (V_1 + V_2 + V_3) = 130 – 80 = 50V$.

c - $R = V_4 / I = 50 / 0.01 = 5\ K\Omega$.

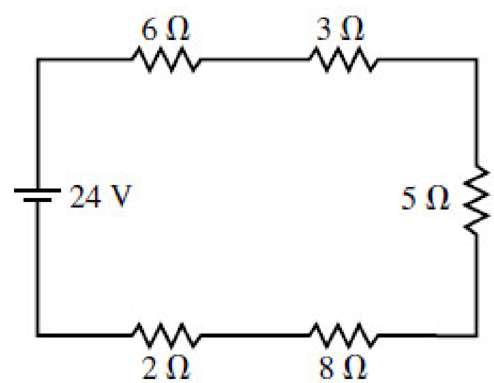
b- $R_T = (3+ 4+ 1+ 5) \times10^3 = 13\ K\Omega$.

e – $P_1 =V_1 \times I = 30\times 0.01 = 0.3\ W = 300\ mW$

$P_2 =V_2 \times I = 40\times 0.01 = 0.4\ W = 400\ mW$

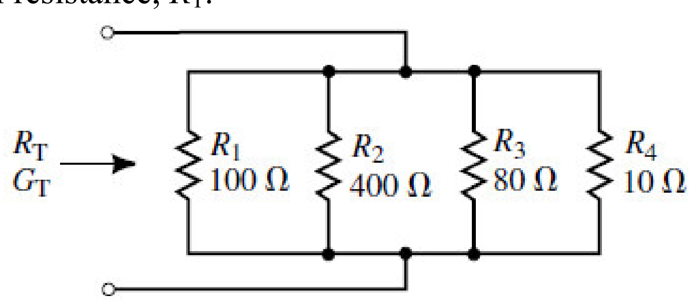
$P_4 = V_4 \times I = 50\times 0.01 = 0.5W = 500\ mW$

Example : Use the voltage divider rule to determine the voltage across each resistor in the circuit of Figure shown below .



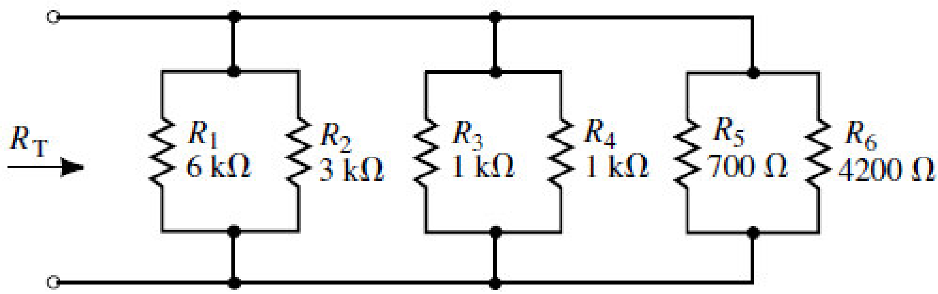
Solution :
$R_T = 6+3+5+8+2 = 24\Omega$.
$V_{6\Omega} = 24\times 6 / 24 = 6V$
$V_{3\Omega} = 24\times 3 / 24 = 3V$
$V_{5\Omega} = 24\times 5 / 24 = 5V$
$V_{8\Omega} = 24\times 8 / 24 = 8V$
$V_{2\Omega} = 24 \times2 / 24 = 2V$

Example : For the parallel network of resistors shown in Figure below, find the total conductance, G_T and the total resistance, R_T.



Solution :
$G_T = G_1 + G_2 + G_3 + G_4$
$= (1/100)+(1/400)+(1/80)+(1/10)$
$= 0.125\ S.$
$R_T = 1/G_T = 8\Omega$.

Example : Find the total equivalent resistance for the network in Figure below.



Solution :

$R_1 // R_2 = 6\times3/6+3 = 2K\Omega$.

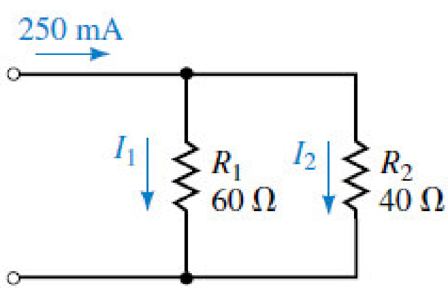
$R_3 // R_4 = 1\times1/1+1 = 500\Omega = 0.5K\Omega$.

$R_5 // R_6 = 700\times4200/700 + 4200 = 600\Omega = 0.6K\Omega$.

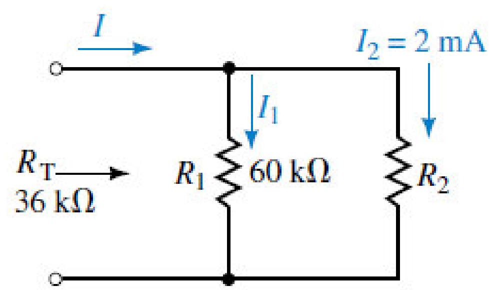
$G_T = (1/2000)+(1/500)+(1/600) = 4.1667\ m\Omega$.

$R_T = 1 / G_T = 240\Omega$.

Example : Use the current divider rule to calculate the unknown currents for the networks of Figure shown below .



a



b

Solution :

a – $I_T = 250$ mA $= 25010^{-3} = 0.25A$.

$I_1 = 0.25\times40 / 100 = 0.1A = 100$ mA

$I_2 = I_T - I_1 = 150$ mA

b - $I_2 = 2mA = 0.002A$, $R_T = 36000\Omega$

$1/36000 = 1/6000\ 0 + 1/R_2$

$R_2 = 90\ K\Omega$.

$0.002 = I \times 60000 / 150000 = I \times 6 / 15$

$I = 0.002\times15 / 6 = 5 \times 10^{-3} = 5mA$

$I_1 = (5 - 2)mA = 3mA$

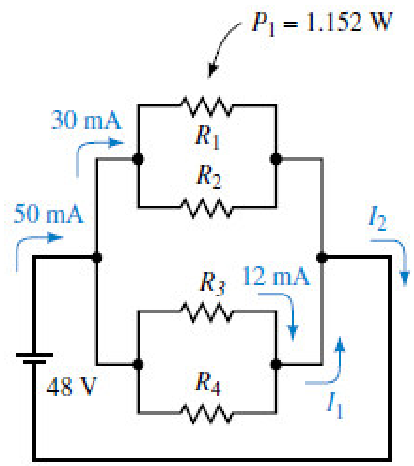
Example : Given the circuit of Figure shown below :

a. Determine the values of all resistors.

b. Calculate the currents through R_1, R_2, and R_4.

c. Find the currents I_1 and I_2.

d. Find the power dissipated by resistors R_2, R_3, and R_4.



Solution :

$E = V_1 = V_2 = V_3 = V_4 = 48V$

$I_T = 50\ mA = I_2$, $30mA = 0.03A, 12mA = 0.012A$

$0.05 = 0.03 + I_1$, $I_1 = 0.02A = 20mA$

b - $P_1 = V_1^2 / R_1$, $1.152 = 2304 / R_1$

$R_1 = 2K\Omega$.

$P_1 = I_{R1}\ V_1$,

$I_{R1} = 1.125/48 = 23.438mA$

$0.03 = 0.023438 + I_{R2}$

$I_{R2} = 6.562mA$

$0.02 = 0.012 + I_{R4}$,

$I_{R4} = 8mA$

c – $I_2 = 50\ mA$, $I_1 = 20mA$

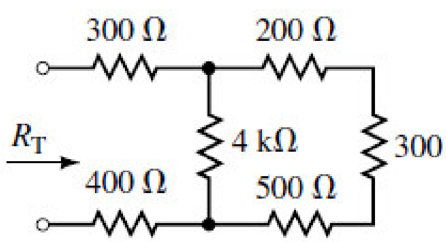
a - $R_1 = 2K\Omega$. , $R_2 = 48 / I_{R2} = 7.314K\Omega$. , $R_3 = 48/ 0.012 = 4K\Omega$., $R_4 = 48/I_{R4} = 6K\Omega$.

d- $P_2 = 48\times 0.006562 = 314.9mW$,

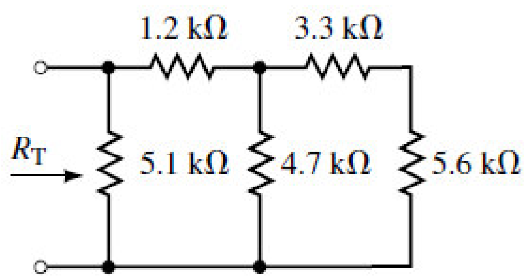
$P_3 = 48 \times 0.012 = 576mW$,

$P = 48 \times 0.008 = 384mW$.

Example : Determine the total resistance of each network in Figure shown below.



a



b

Solution :

a-$R_T = 300+(4000//(200+300+500))+400 = 300+400 +(4000\times1000/5000) = 1500\Omega$.

b-$R_T = 5.1K//(1.2K+(4.7K//(3.3K+5.6K))) = 2.33K\Omega$.

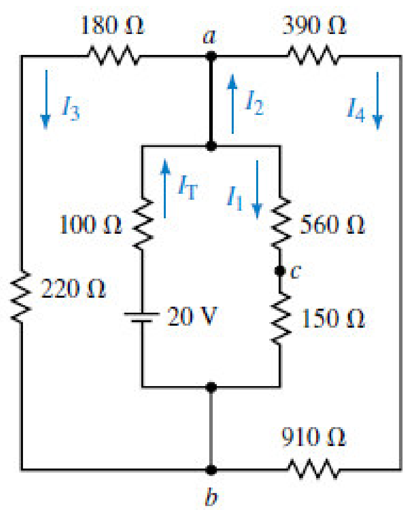
Example : For the circuit of Figure shown below :

Find the following quantities:

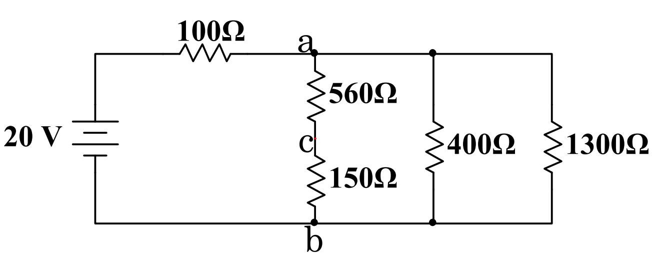
a. R_T

b. I_T, I_1, I_2, I_3, I_4

c. V_{ab}, V_{bc}.



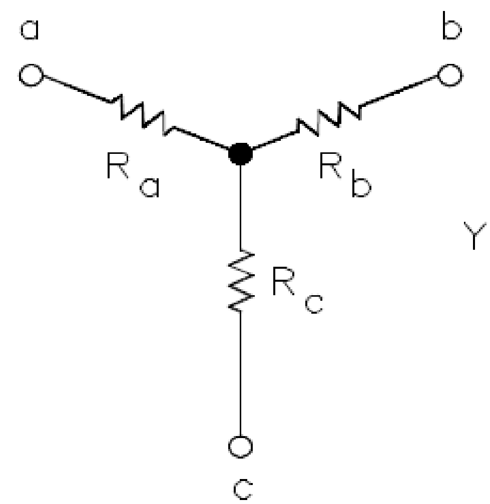
Solution : The equivalent circuit



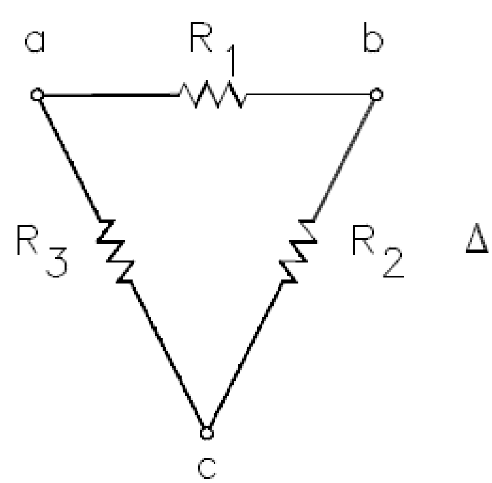
a-R_T = 100 + (710×400×1300/(710×400)+(710×1300)+(400×1300)) = 314Ω.
c- V_{ab} = 20 ×214 / 314 = 13.63V.

b- I_T = 20/314 = 0.0637A=63.7mA
$I_1 = V_{ab}$ / 710 = 0.0192A=19.2mA
$I_2 = I_T - I_1$ =44.5 mA
$I_3 = V_{ab}$ / 400 = 0.034A =34.1 mA
$I_4 = V_{ab}$ / 1300 = 0.01048A = 10.48mA
V_{bc} = - 0.0192× 150 = -2.9V

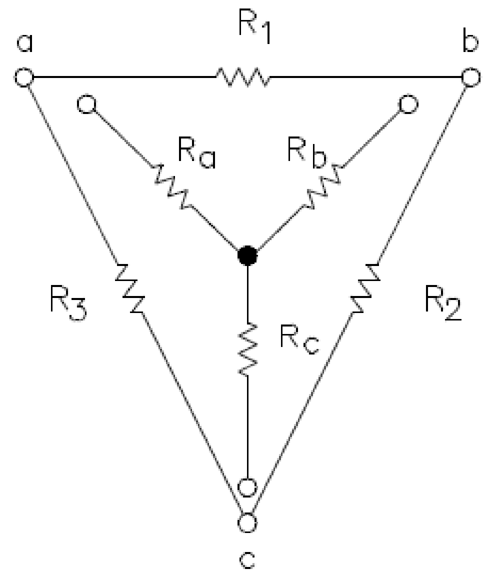
2-3 Star (Y)-Delta(Δ) Connection
Because of its shape, the network shown in Figure below is called a T (tee) or Y (wye) network.These are different names for the same network .



The network shown in Figure below is called p (pi) or D (delta) because the shapes resemble Greek letters p and W. These are different names for the same network.



• Delta -Star Conversion

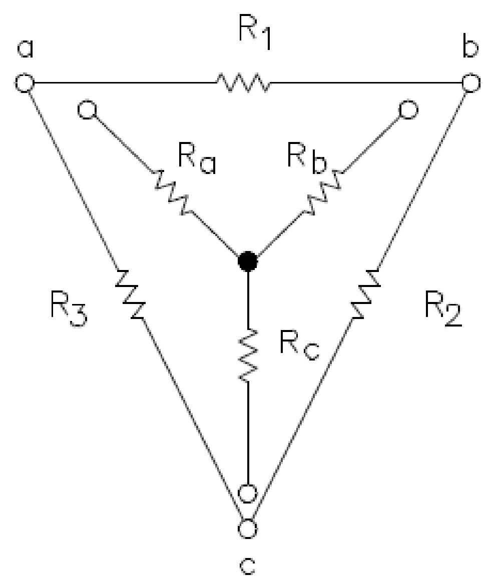


$R_a = (R_1 R_3) / (R_1+R_2+R_3)$

$R_b = (R_1 R_2) / (R_1+R_2+R_3)$

$R_c = (R_2 R_3) / (R_1+R_2+R_3)$

• Star -Delta Conversion



$R_1 = (R_a R_b+R_b R_c+R_c R_a) / R_c$

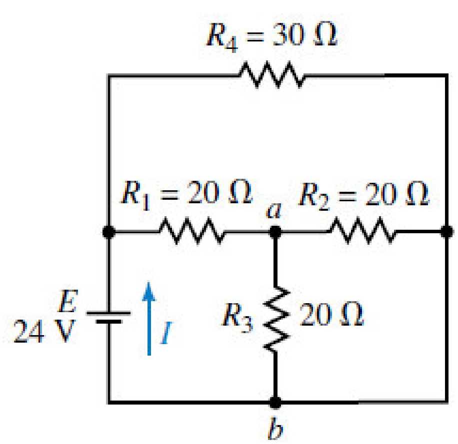
$R_2 = (R_a R_b+R_b R_c+R_c R_a) / R_a$

$R_3 = (R_a R_b+R_b R_c+R_c R_a) / R_b$

Example : Using Δ-Y or Y-Δ conversion, find the current I and the voltage V_{ab} for the circuit of Figure shown below .

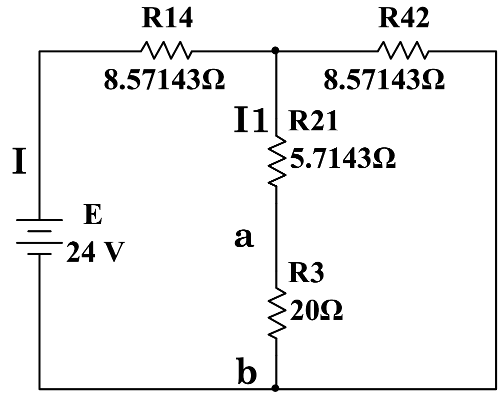
Solution :

We may convert the "Δ" into its equivalent "Y" Δ its(R_1, R_2,and R_4).

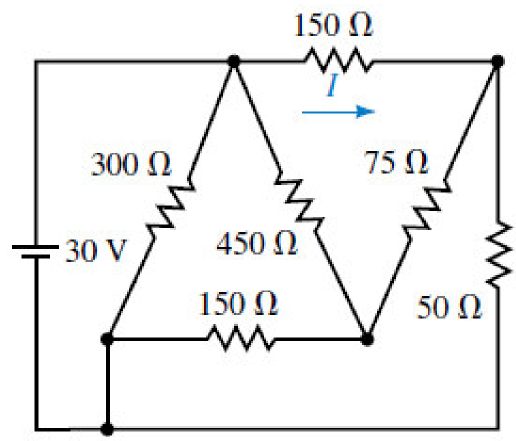


$R_{14} = (20\times30) / 20+30+20 = 8.57143\Omega$
$R_{42} = (30\times20) / 20+30+20 = 8.57143\Omega$
$R_{21} = (20\times20) / 20+30+20 = 5.7143\Omega$

The equivalent circuit
$R_T = 8.57143 + 6.429 = 15\ \Omega$
$I = 24 / 15 = 1.6$ A.
$I_1 = 1.6 \times 8.57143 / 34.29 = 0.4$A
$V_{ab} = 0.4 \times 20 = 8$V.



Example : Using Δ-Y orY-Δ conversion, find the current *I* for the circuit of Figure shown below .

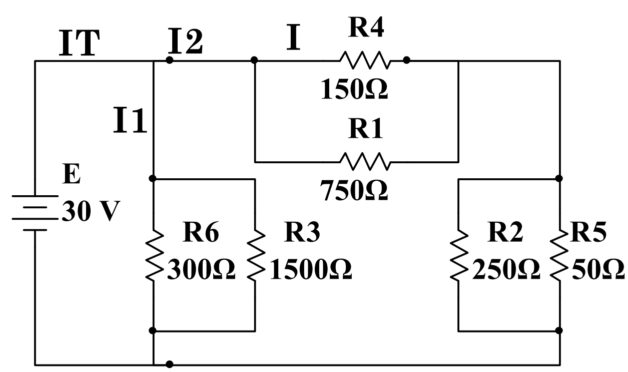


Solution :

we may convert the "Y" into its equivalent "Δ".
Y its (450 ,75 and 150)
$R_1 = (450\times75 + 75\times150 + 150\times450) / 150 = 112500/150 = 750\Omega$
$R_2 = (450\times75 + 75\times150 + 150\times450) / 450 = 112500/450 = 250\Omega$
$R_3 = (450\times75 + 75\times150 + 150\times450) / 75 = 112500/75 = 1500\Omega$

The equivalent circuit
$R_6 // R_3 = 250\Omega$
$R_4 // R_1 + R_5 // R_2 = 125 + 41.67 = 166.67\Omega$

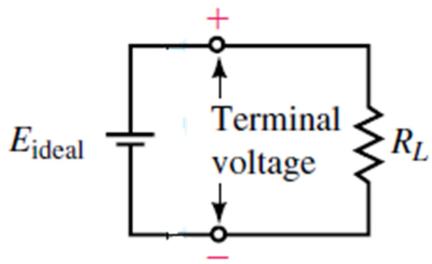
$R_T = 250 // 166.67 = 100\Omega$
$I_T = 30/100 = 0.3$A
$I_2 = 0.3\times250 / (250 + 166.67)$
$= 0.18$A
$I = 0.18\times750 / 900 = 0.149$A.



2-4 Sources of Energy

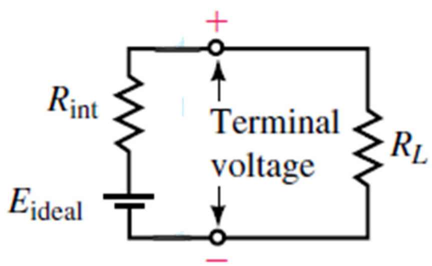
a - Ideal voltage source:

It is a device that provides a constant voltage across its terminals ($V_L = E$) whatever the drawn current is . It does not exist practically.



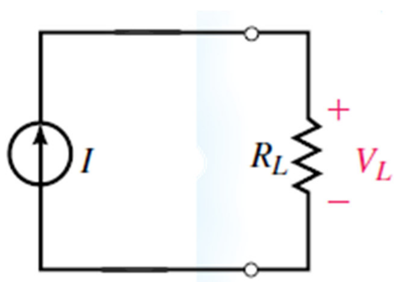
-Actual or Practical Voltage Source:

it is device with an internal resistance and can be represented by a voltage source in series with an internal resistance as shown below.

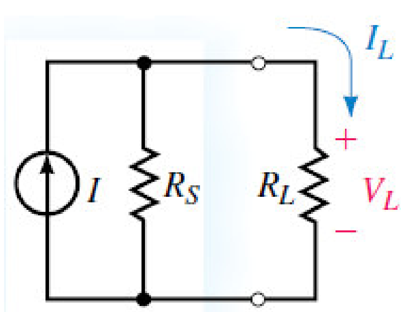


b- ideal current source:

It is a device that provides a constant current ($I_L = I$) to any load resistance connected across it independent of the voltage across its terminals.

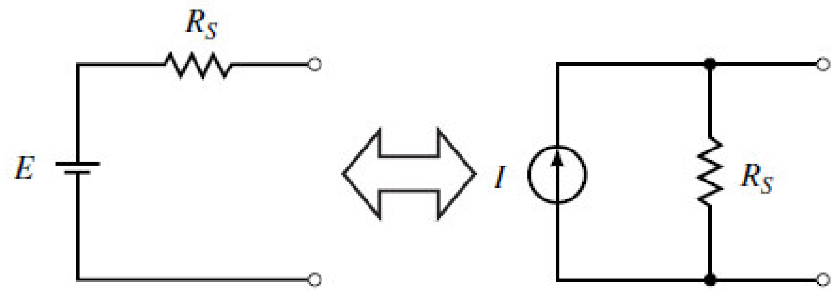


-practical current source:



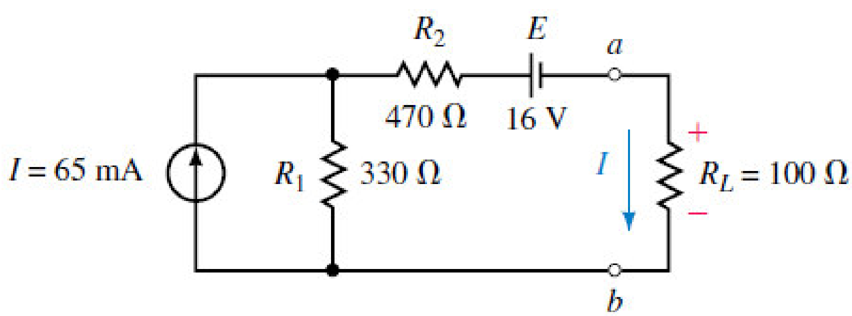
2-5 Source Conversions

voltage source can be transformed to current source and vice versa as follows:

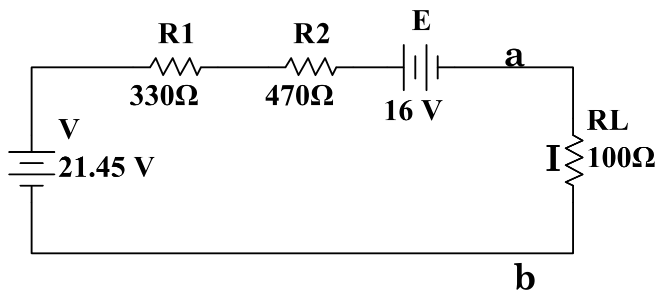


$E = I\,R_s$ $\qquad$ $I = E / R_s$

Example : Convert the current source into its equivalent voltage source and Find V_{ab} and I for the network of Figure shown below.



Solution :



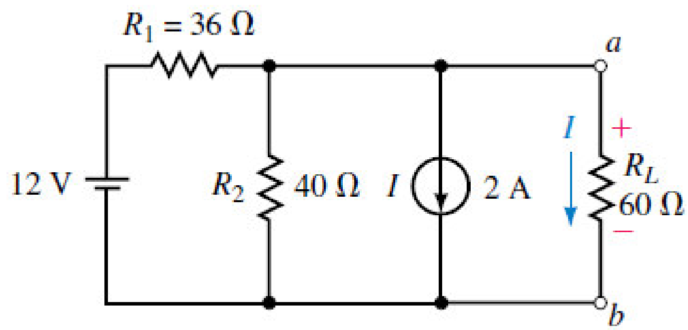
$V = I \times 330 = 0.065 \times 330 = 21.45\ V$

$900 \times I + 21.45 - 16 = 0$

$I = 5.45/900 = 0.0060556A = 6.06mA$

$V_{ab} = I \times 100 = 0.60556V$

Example : Convert the voltage sourc and the 36Ω resistor into an equivalent current source ,and determine the current I through $R_L$and voltage V_{ab} .



Solution :

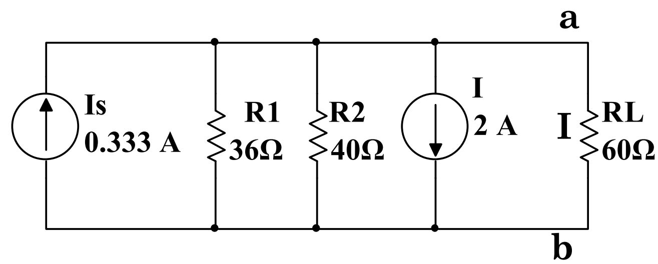
$I_S = 12 / 36 = 0.333A$

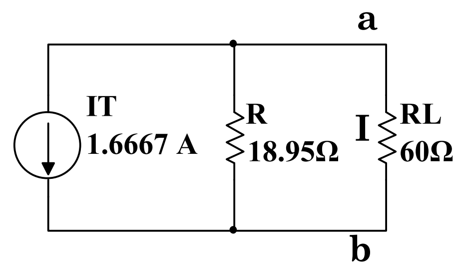
$R = 36 \times 40 / 76 = 18.95\Omega$

$I_T = 2 - 0.333 = 1.6667A$

$I = I_T \times 18.95 / 78.95 = 0.4\ A$

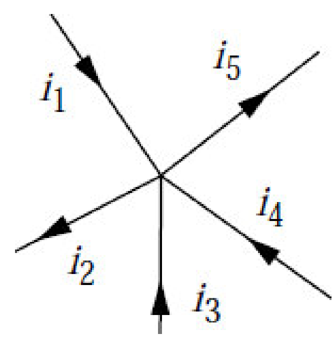
$V_{ab} = 0.4 \times 60 = 24\ V$





2-6 Kirchhoff's Laws

a-Kirchhoff's current law (KCL) states that the currents at any node algebraically sum to zero. In other words, the sum of the currents entering a node equals the sum of currents leaving the node.



$\Sigma I_{in} = \Sigma I_{out}$

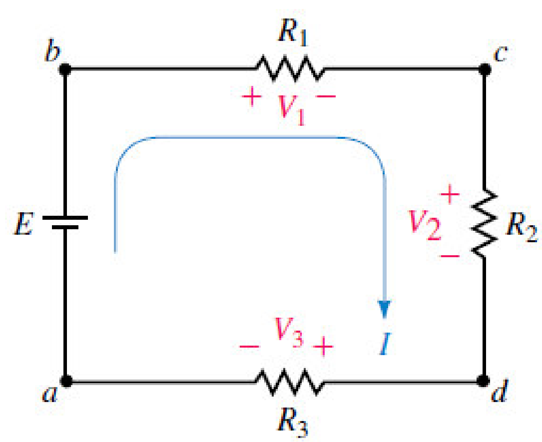
$I_1 + I_3 + I_4 = I_2 + I_5$

or

$\sum_{n=1}^{N} I n = 0$

$I_1 - I_2 + I_3 + I_4 - I_5 = 0$

b- Kirchhoff's voltage law (KVL) states that the voltages around aclosed path algebraically sum to zero. In other words, the sum of voltage rises equals the sum of voltage drops.



Sum of voltage drops = Sum of voltage rises

$\Sigma E_{rises} = \Sigma V_{drops}$ for a closed loop

$E = V1 + V2 + V3$

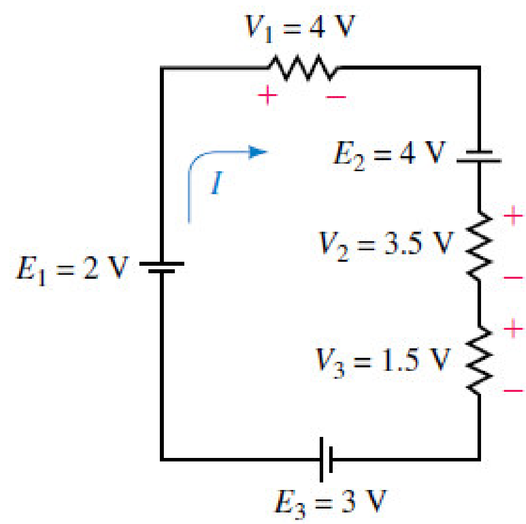
or

$\Sigma V = 0$ for a closed loop

$E - V1 - V2 - V3 = 0$ or $V_3 + V_2 + V_1 - E = 0$ in the opposite direction

Example : Verify Kirchhoff's voltage law for the circuit of Figure shown below .

Solution :



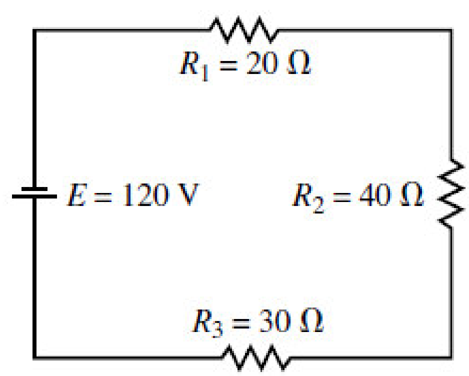
$E_1 - V_1 + E_2 - V_2 - V_3 + E_3 = 0$

$2\ V - 4\ V + 4\ V - 3.5\ V - 1.5\ V + 3\ V = 0$

or

$E_1+E_2+E_3=V_1+V_2+V_3$

Example : For the series circuit shown in Figure below, find the following quantities:



a. Total resistance, R_T.

b. The direction and magnitude of the current, I.

c. Polarity and magnitude of the voltage across each resistor.

d. Power dissipated by each resistor.

e. Power delivered to the circuit by the voltage source.

f. Show that the power dissipated is equal to the power delivered.

Answers:

a. 90.0 Ω

b. 1.33 A counterclockwise

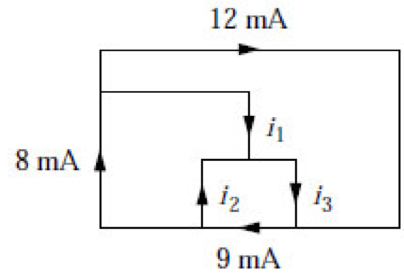
c. V1 = 26.7 V , V2 = 53.3 V , V3 = 40.0 V

d. P1 = 35.6 W , P2 = 71.1 W , P3 = 53.3 W.

e. PT = 160. W

f. P1 + P2 + P3 = 160. W =PT

Example : Use KCL to obtain currents I1, I2, and I3 in the circuit shown in Fig.
Solution :



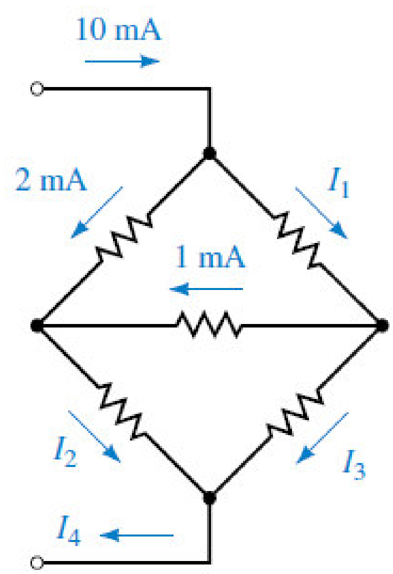
$I_3 = I_1 + I_3$

$8mA = 12mA + I_1$

$9mA = 8mA + I_2$

$I_1 = -4mA,\ I_2 = 1mA, I_3 = -3mA.$

Example : Find each of the unknown currents in the networks of Figure.
Solution :



$10mA = 2mA + I_1$

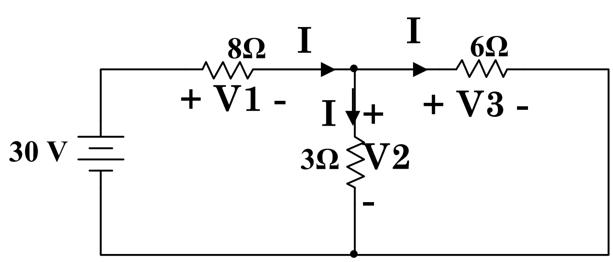
$I_2 = 2mA + 1mA$

$I_1 = 1mA + I_3$

$I_4 = I_2 + I_3$

$I_1 = 8mA$, $I_2 = 3mA$, $I_3 = 7mA$, $I_4 = 10mA$.

Example : Find the currents and voltages in the circuit shown in Fig.



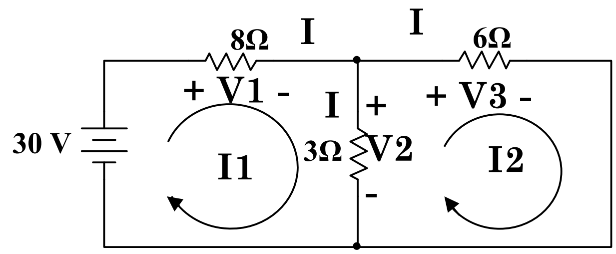
Solution : We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$V_1 = 8\ I_1$, $V_2 = 3\ I_2$, $V_3 = 6\ I_3$

A-At node *a*, KCL gives

$I_1 = I_2 + I_3$

B-Applying KVL to loop 1 as in Fig.below.



$30 - V_1 - V_2 = 0$

$30 - 8I_1 - 3I_2 = 0$

$I_1 = (30 - 3I_2)/8$

C-Applying KVL to loop 2,

$V_2 - V_3 = 0$

$3I_2 - 6I_3 = 0$

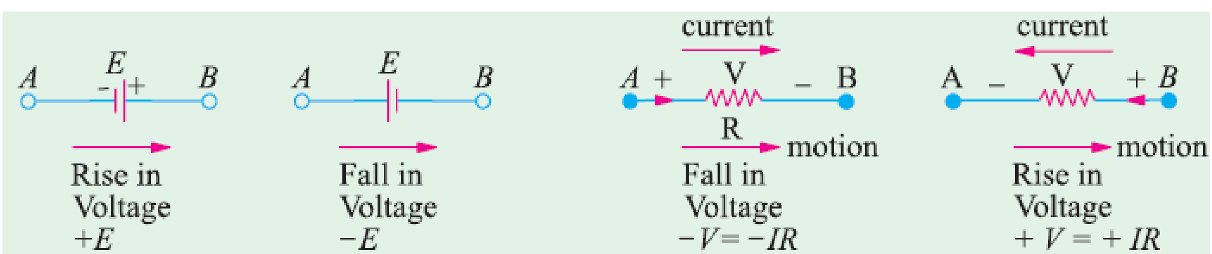
$V_2 = V_3 \Longrightarrow 3I_2 = 6I_3 \Longrightarrow I_3 = I_2 / 2$

Substituting Eqs. (A) and (B) into (C) gives

$(30 - 3I_2)/8 = I_2 + I_2 / 2$

$30 - 3I_2 = 8I_2 + 4I_2$

$30 = 15I_2 \Longrightarrow I_2 = 2A \;,\; I_1 = 3A \;,\; I_3 = 1 A.$



Chapter Three
Methods of Analysis

3-1 Maxwell's Loop Current Method

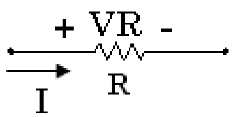
The following procedures must be carried to apply this method :

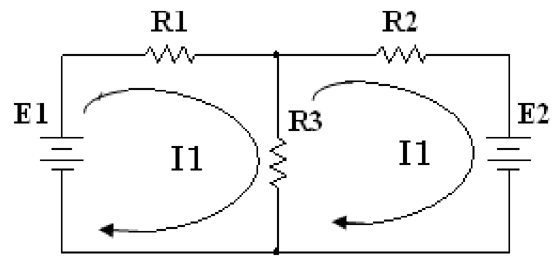
1- currents are assumed to flow clockwise around the loop through the parameters of the loop with out splitting at the junctions.

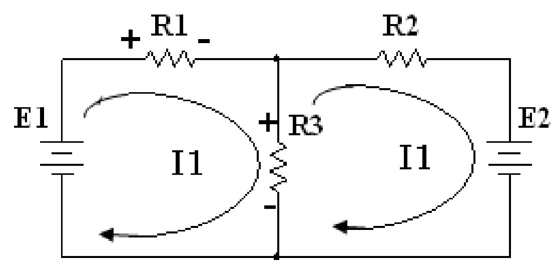
2- kirchhoff's voltage law applied around each loop .

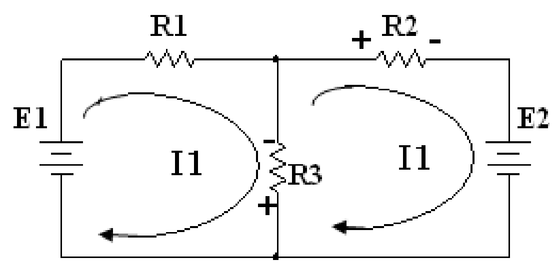
3- solve the resultant simultaneous linear equation for the assumed currents .

In this Fig., for example, The current I, defined as flowing from left to right, establishes the polarity of the voltage across *R*.









For loop 1 :

$E_1 - I_1 R_1 - I_3 R_3 = 0$

$I_3 = I_1 - I_2$

$E_1 = I_1 R_1 + (I_1 - I_2)R_3$

• $I_1 (R_1 + R_3) - I_2 R_3 = E_1$

For loop 2 :

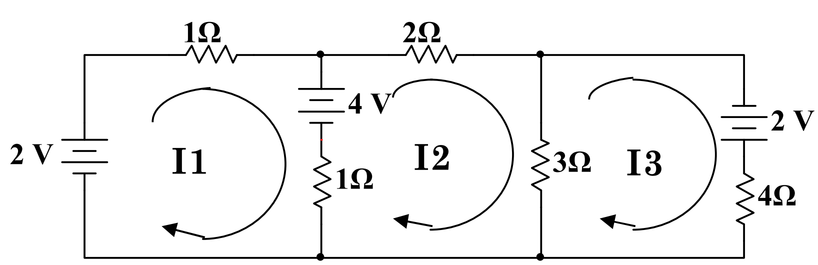
$-E_2 + I_3 R_3 + I_2 R_2 = 0$

$I_3 = I_2 - I_1$

$-E_2 = -(I_2 - I_1) R_3 - I_2 R_2$

• $-I_1 R_3 + I_2 (R_2 + R_3) = -E_2$

Example : Write the loop equations for the network shown in fiqure below .



Solution :

For loop 1 :

$(1+1)I_1 - 1I_2 - 0I_3 = 2 - 4$

$2I_1 - 1I_2 = -2$

For loop 2 :

$-I_1 + (1+2+3)I_2 - 4I_3 = 4$

$-I_1 + 6I_2 - 4I_3 = 4$

For loop 3 :

$-0I_1 - 3I_2 + (3+4)I_3 = 2$

$-3I_2 + 7I_3 = 2$

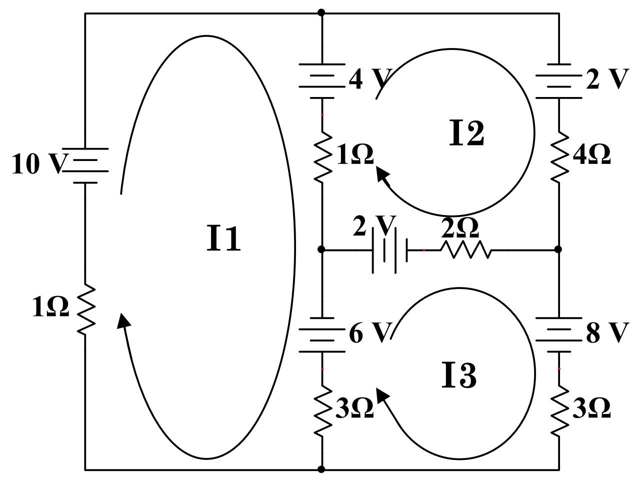
Loop equations are as following :

$2I_1 - 1I_2 = -2$ ………………1

$-I_1 + 6I_2 - 4I_3 = 4$ …………2

$-3I_2 + 7I_3 = 2$ ………………3

Example : Using Maxwell's loop current method find the current in the 2Ω resistor in the network below .



Solution :

For loop 1:

$5I_1 - 1I_2 - 3I_3 = 12$ ……1

For loop 2:

$-1I_1 + 7I_2 - 2I_3 = 8$ ……2

For loop 3 :

$-3I_1 - 2I_3 + 8I_3 = -16$…..3

To use Cramer's rule, we cast Eqs. (1) ,(2) and(3) in matrix form as

$\Delta\, I = V$, $I_1=\Delta_1/\Delta$, $I_2=\Delta_2/\Delta$, $I_3=\Delta_3/\Delta$

$$\begin{bmatrix} 5 & -1 & -3 \\ -1 & 7 & -2 \\ -3 & -2 & 8 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ -16 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -1 & -3 \\ -1 & 7 & -2 \\ -3 & -2 & 8 \end{vmatrix} \begin{matrix} 5 & -1 \\ -1 & 7 \\ -3 & -2 \end{matrix} = ((5\times7\times8)+(-1\times-2\times-3)+(-3\times-1\times-2))$$

$$-((-3\times7\times-3)+(5\times-2\times-2)+(-1\times-1\times8)) = 177$$

$$\Delta 1 = \begin{vmatrix} 12 & -1 & -3 \\ 8 & 7 & -2 \\ -16 & -2 & 8 \end{vmatrix} \begin{matrix} 12 & -1 \\ 8 & 7 \\ -16 & -2 \end{matrix} = 368$$

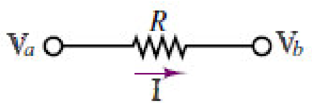
$I_1 = \Delta 1/\Delta = 368/177 = 2.079$ A.
By using same steps
$I_2 = 1.175$ A , $I_3 = -0.9265$ A.

3-2 Nodal Analysis

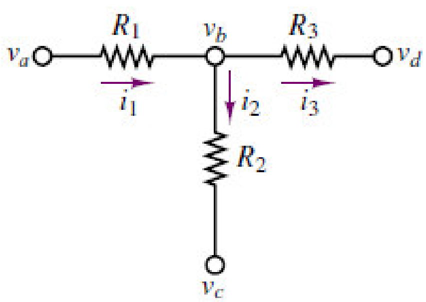
Node Voltage Analysis Method

1. Select a reference node(usually ground). All other node voltages will be referenced to this node.
2. Define the remaining $n-1$ node voltages as the independent variables.
3. Apply KCL at each of the $n-1$ nodes, expressing each current in terms of the adjacent node voltages.
4. Solve the linear system of $n-1$ equations in $n-1$ unknowns.

- In the node voltage method, we assign the node voltages Va and Vb; the branch current flowing from a to b is then expressed in terms of these node voltages.



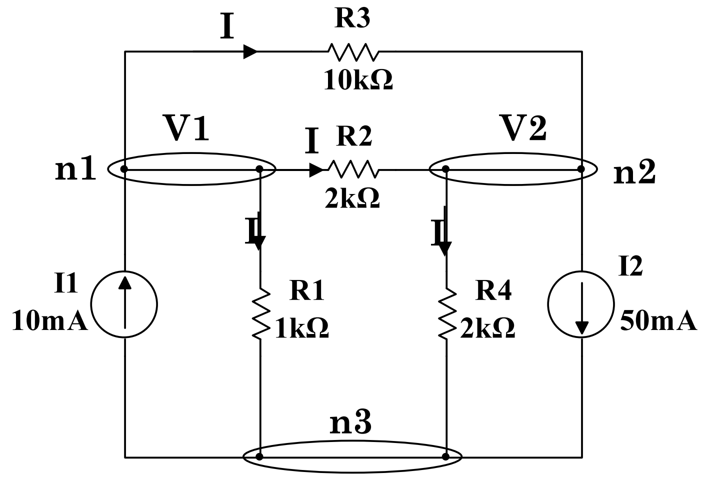
$I = (V_a - V_b) / R$



By KCL: $I_1 = I_2 - I_3$. In the node voltage method, we express KCL by

$$(V_a - V_b) / R_1 = (V_b - V_c)/R_2 + (V_b - V_d)/R_3$$

Example : For the circuit of Figure shown below find all node voltages and branch currents. Schematics, diagrams, circuits, and given data:
$I_1 = 10$ mA ; $I_2 = 50$ mA ; $R_1 = 1$ kΩ ; $R_2 = 2$ kΩ ; $R_3 = 10$ kΩ ; $R_4 = 2$ kΩ .



Solution : In this circuit three nodes , Note that we have selected to ground the lower part of the circuit, resulting in a reference voltage of zero at that node.
V_1 : at node 1
V_2 : at node 2

Applying KCL at nodes 1 and 2 we obtain

$I_1 - (V_1 - 0)/R_1 - (V_1 - V_2)/R_2 - (V_1 - V_2)/R_3 = 0$ (at node 1)

$(V_1 - V_2)/R_2 + (V_1 - V_2)/R_3 - (V_2 - 0)/R_4 - I_2 = 0$ (at node 2)

With some manipulation, the equations finally lead to the following form:

$$1.6V_1 - 0.6V_2 = 10$$
$$-0.6V_1 + 1.1V_2 = -50$$

$$\begin{bmatrix} 1.6 & -0.6 \\ -0.6 & 1.1 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \end{bmatrix} = \begin{bmatrix} 10 \\ -50 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 1.6 & -0.6 \\ -0.6 & 1.1 \end{bmatrix} = 1.4$$

$$\Delta 1 = \begin{bmatrix} 10 & -0.6 \\ -50 & 1.1 \end{bmatrix} = -19 \quad \Longrightarrow \quad V_1 = -19 / 1.4 = -13.57V$$

$$\Delta 2 = \begin{bmatrix} 1.6 & 10 \\ -0.6 & -50 \end{bmatrix} = -74 \quad \Longrightarrow \quad V_2 = -74 / 1.4 = -52.875V.$$

$I_{10k\Omega} = V_1 - V_2/10000 = 3.93$ mA.
$I_{1k\Omega} = V_1 /1000 = -13.57$ mA.
$I_{2k\Omega} = V_1 - V_2/2000 = 19.65$ mA.
$I_{R4} = V_2/2000 = -26.4375$ mA.

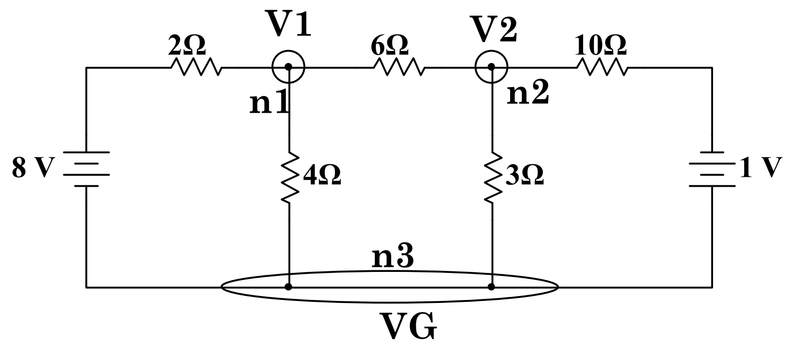
- Now we can write the same equations more systematically as a function of the unknown node voltages, as was done in terms of the conductances .

$(G_1+G_2 + G_3) V_1 - (G_2+G_3) V_2 = I_1$ (at node 1)
$-(G_2+G_3) V_1 + (G_2+G_3 + G_4) V_2 = - I_2$ (at node 2)

$$\left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}\right) V1 - \left(\frac{1}{R2} + \frac{1}{R3}\right) V2 = I_1$$

$$-\left(\frac{1}{R2} + \frac{1}{R3}\right) V1 + \left(\frac{1}{R2} + \frac{1}{R3} + \frac{1}{R4}\right) V2 = -I_2$$

Example:Find the voltage across the 3Ω resisror of the network shown below ;



Solution :

For node 1:

$$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)V1-\left(\frac{1}{6}\right)V2=\frac{8}{2} \Rightarrow 0.9167V_1-0.167V_2=4$$

For node 2 :

$$-\left(\frac{1}{6}\right)V1+\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{10}\right)V2=-\frac{1}{10} \Rightarrow -0.167V_1+0.6V_2=-0.1$$

$$V_{3\Omega}=V_2=\frac{\begin{bmatrix}0.9167 & 4\\ -0.167 & -0.1\end{bmatrix}}{\begin{bmatrix}0.9167 & -0.167\\ -0.167 & 0.6\end{bmatrix}}=\frac{-0.09267+0.668}{0.55-0.0279}=1.101V$$

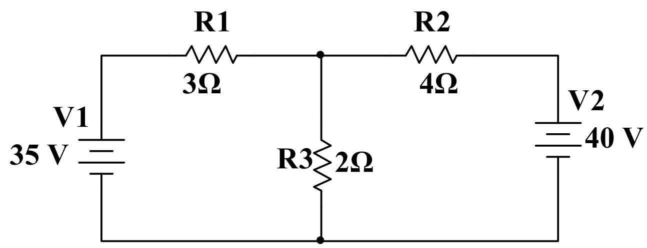
3-3 Superposition Theorem

The superposition principle states that for a circuit having multiple sources, the voltage across (or current through) an element is equal to the algebraic sum of all the individual voltages (or currents) due to each source acting one at a time.

Steps to Apply Superposition Principle :

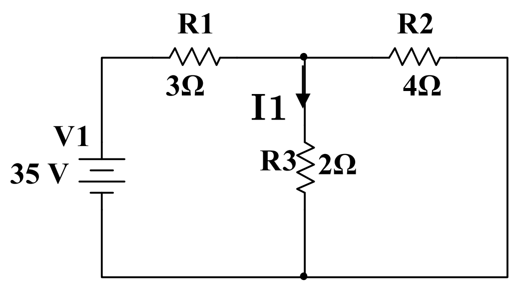
1. Turn off all sources except one source. Find the output (voltage or current) due to that active source using nodal or any other theorem analysis.
2. Repeat step 1 for each of the other sources.
3. Find the total contribution by adding algebraically all the contributions due to the sources.

Example : In the network shown below , find the current in the branch 2Ω by superposition theorem.



Solution :

a- source V_2 is short- circuited.

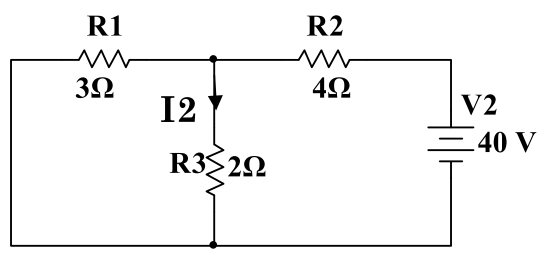


$R_{eq} = 3 + (2\times4/2+4) = 3+(8/6) = 4.333\Omega$

$I = 35 / 4.333 = 8.077A$

$I_1 = 8.077\times 4 / 6 = 5.3846A.$

b- source V_1 is short- circuited.



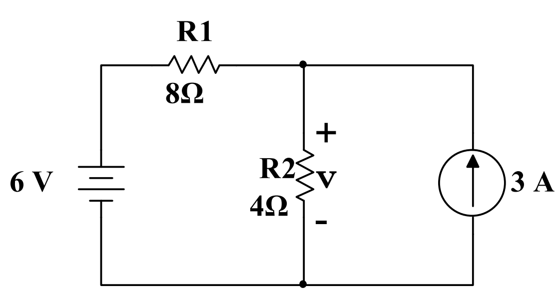
$R_{eq} = 4 + (6/5) = 5.2\Omega$

$I = 40 / 5.2 = 7.692A$

$I_2 = 7.692 \times 3 / 5 = 4.615A$

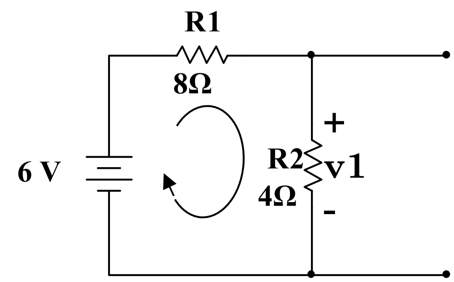
$I_{2\Omega} = I_1 + I_2 = 5.3846 + 4.615 = 10A.$

Example : Use the superposition theorem to find V in the circuit in Fig.shown below.



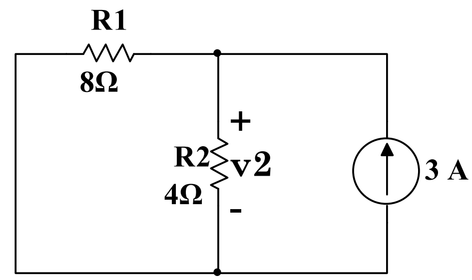
Solution :
Since there are two sources, let

$$V = V_1 + V_2$$



a- source 3A is open- circuited.

$I = 6/12 = 0.5A$
$V_1 = 0.5 \times 4 = 2V.$

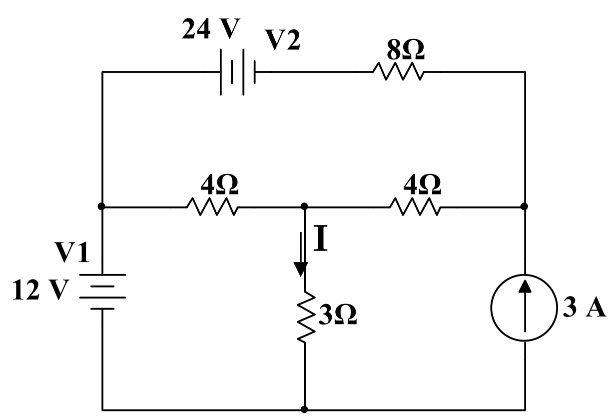


b- source 6V is short- circuited.

$I_{4\Omega} = 3\times8 / 12 = 2A$
$V_2 = 2\times 4 = 8V$

- $V = V_1 + V_2 = 10V.$

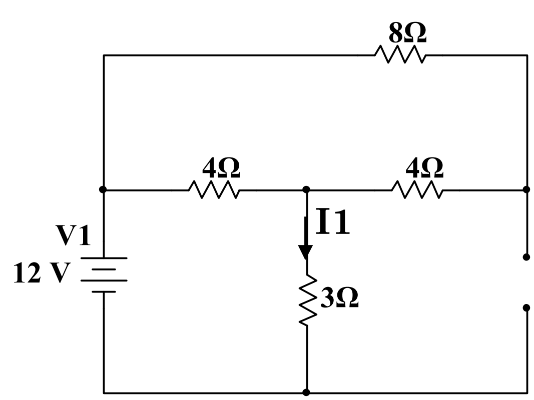
Example: For the circuit in Fig. shown below, use the superposition theorem to find I.



Solution:
In this case, we have three sources. Let

$I = I_1 + I_2 + I_3$

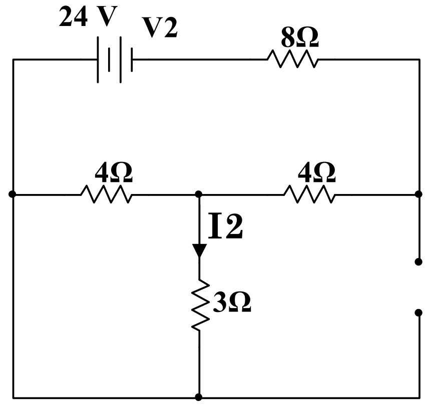
a- source 24V is short- circuited and source 3A is open- circuited.



$8\Omega + 4\Omega // 4\Omega = 3\Omega$

$I_1 = 12 / 6 = 2A.$

b- source 12V is short- circuited and source 3A is open- circuited.

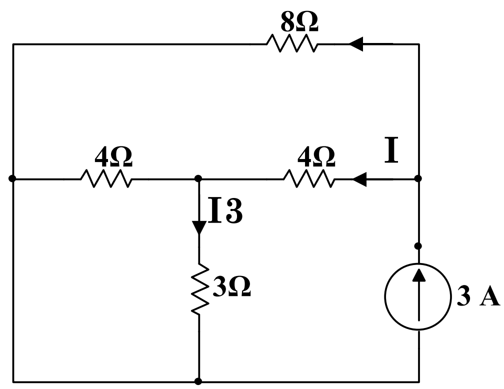


$4\Omega \,//\, 3\Omega = 1.7145\Omega$

$V = -24 \times 1.7145 / 13.7145 = -3V$

$I_2 = 3/3 = -1A.$

c - source 12V is short- circuited and source 24V is short- circuited.



$4\Omega \,//\, 3\Omega = 1.7143\Omega$
$4 + 1.7143 = 5.7143\Omega$

$I = 3\times8 / (5.7143+8) =1.75\ A$
$I_3 = 1.75\times 4 / 7 = 1A.$

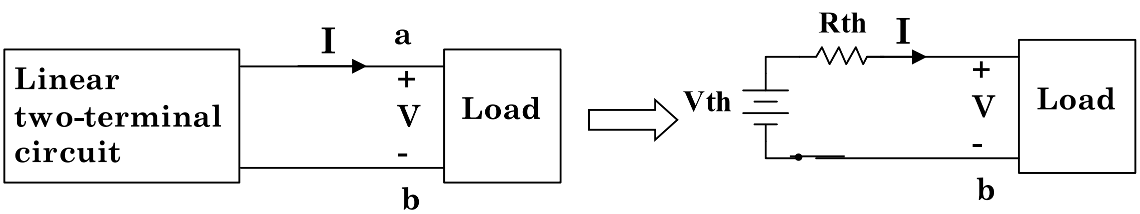
• $I = I_1 + I_2 + I_3 = 2 – 1 + 1 = 2A.$

3- 4 Thevenin's Theorem

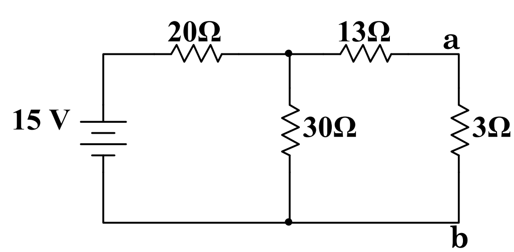
Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

The following steps provide a technique which converts any circuit into its Thévenin equivalent:

1. Remove the load from the circuit.
2. Label the resulting two terminals. We will label them as *a* and *b,* although any notation may be used.
3. Set all sources in the circuit to zero.
   Voltage sources are set to zero by replacing them with short circuits (zero volts).
   Current sources are set to zero by replacing them with open circuits (zero amps).
4. Determine the Thévenin equivalent resistance, *R*Th, by calculating the resistance "seen" between terminals *a* and *b*. It may be necessary to redraw the circuit to simplify this step.
5. Replace the sources removed in Step 3, and determine the open-circuit voltage between the terminals. If the circuit has more than one source, it may be necessary to use the superposition theorem. In that case, it will be necessary to determine the open-circuit voltage due to each source separately and then determine the combined effect. The resulting open-circuit voltage will be the value of the Thévenin voltage,E_{Th}.
6. Draw the Thévenin equivalent circuit using the resistance determined in Step 4 and the voltage calculated in Step 5. As part of the resulting circuit,include that portion of the network removed in Step 1.



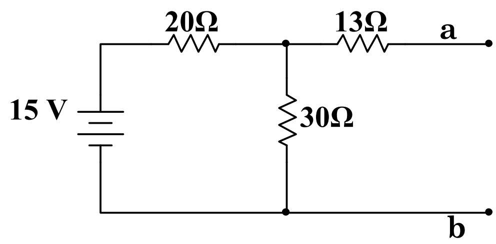
Example : Find Thévenin equivalent circuit for the following circuit between points a and b .



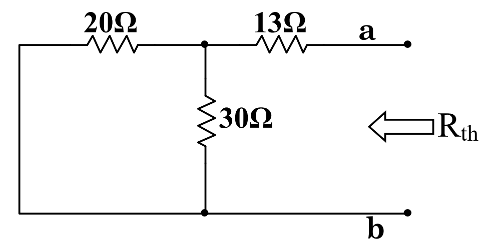
Solution :

Remove the load.

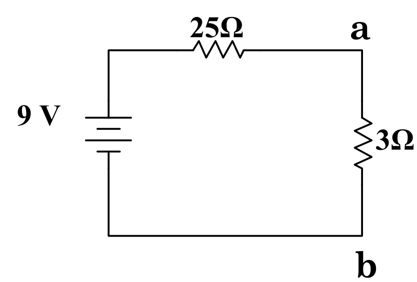
$V_{th} = 15\times30 / 50 = 9V$



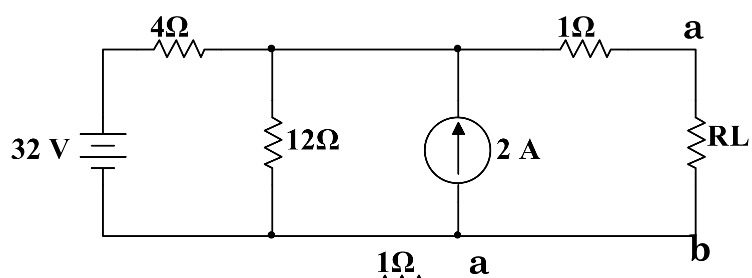
$R_{th} = 13 + (30//20) = 25\Omega$



Thévenin equivalent circuit



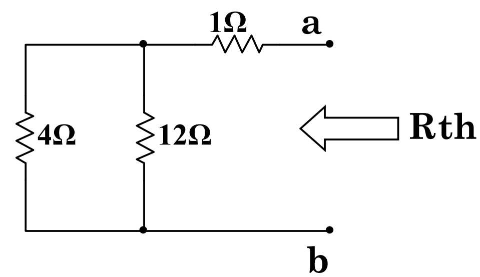
Example : Find the Thevenin equivalent circuit of the circuit shown in Fig. shown below, to the left of the terminals *a*-*b*. Then find the current through R_L = 6Ω, 16Ω, and 36 Ω.



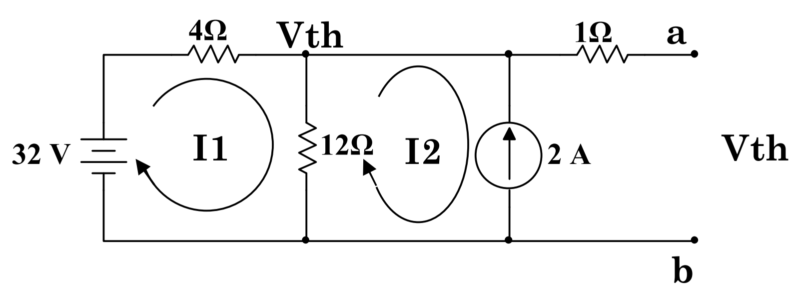
Solution :

Remove the load.

$R_{Th} = (4 \ // \ 12) + 1 = 4\Omega$



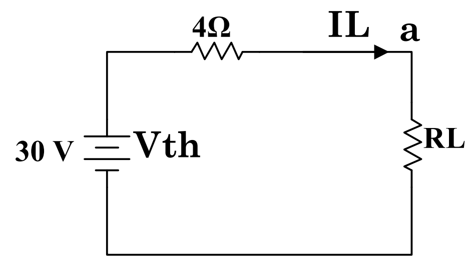
source 32V is short- circuited and source 2A is open- circuited.
To find V_{Th}, consider the circuit in Fig. shown below. Applying maxwell's analysis to the two loops, we obtain



$16I_1 - 12I_2 = 32$
$I_2 = -2$
$16I_1 + 24 = 32$, $16 I_1 = 8$, $I_1 = 8/16 = 0.5A.$

$V_{th} = 12(I_1 - I_2) = 12 (0.5 + 2) = 30V.$

The Thevenin equivalent circuit is shown in Fig. shown below. The current through R_L is



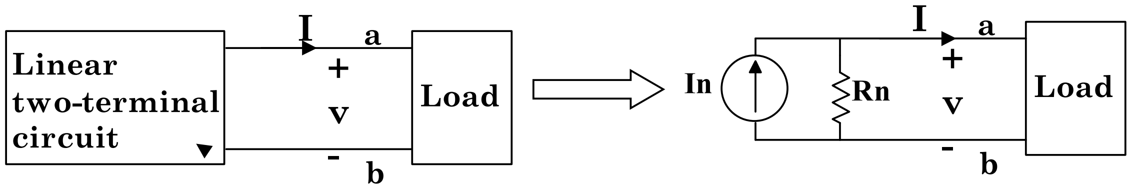
•When $R_L = 6\Omega$,
$I_L = V_{th} / (R_{th} + R_L) = 30 / 10 = 3A.$

•When $R_L = 16\Omega$,
$I_L = V_{th} / (R_{th} + R_L) = 30 / 20 = 1.5A.$

•When $R_L = 36\Omega$,
$I_L = V_{th} / (R_{th} + R_L) = 30 / 40 = 0.75A.$

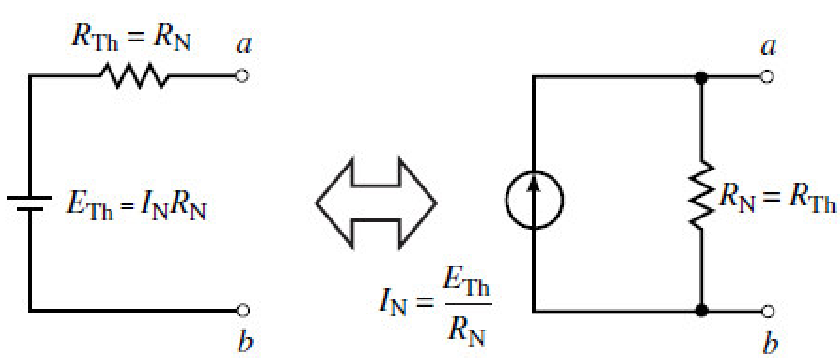
3-5 Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



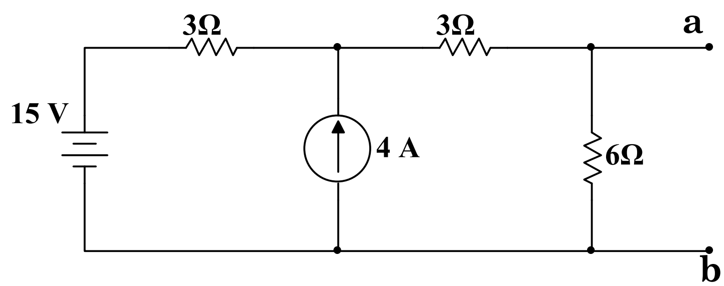
The following steps provide a technique which allows the conversion of any circuit into its Norton equivalent:

1. Remove the load from the circuit.
2. Label the resulting two terminals. We will label them as *a* and *b,* although any notation may be used.
3. Set all sources to zero. As before, voltage sources are set to zero by replacing them with short circuits and current sources are set to zero by replacing them with open circuits.
4. Determine the Norton equivalent resistance, *R*N, by calculating the resistance seen between terminals *a* and *b.* It may be necessary to redraw the circuit to simplify this step.
5. Replace the sources removed in Step 3, and determine the current which would occur in a short if the short were connected between terminals *a* and *b.* If the original circuit has more than one source, it may be necessary to use the superposition theorem. In this case, it will be necessary to determine the short-circuit current due to each source separately and then determine the combined effect. The resulting short-circuit current will bethe value of the Norton current *I*N.
6. Sketch the Norton equivalent circuit using the resistance determined in Step 4 and the current calculated in Step 5. As part of the resulting circuit, include that portion of the network removed in Step 1.



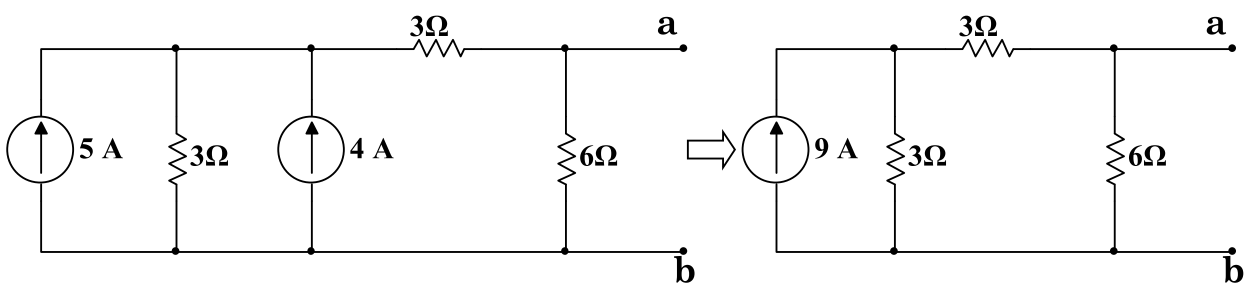
Thévenin equivalent circuit Norton equivalent circuit

Example : Find the Norton equivalent circuit for the circuit in Fig.shown below.

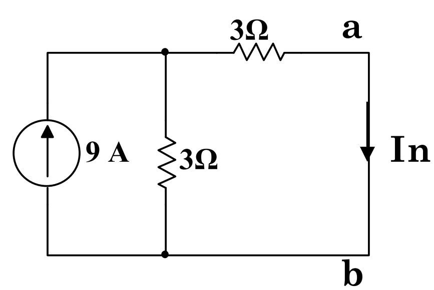


Solution :

By using conversion voltage source to current source

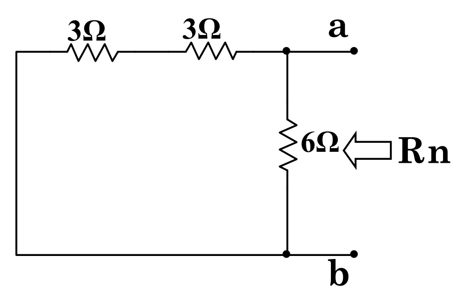


the short connection between terminals a and b.

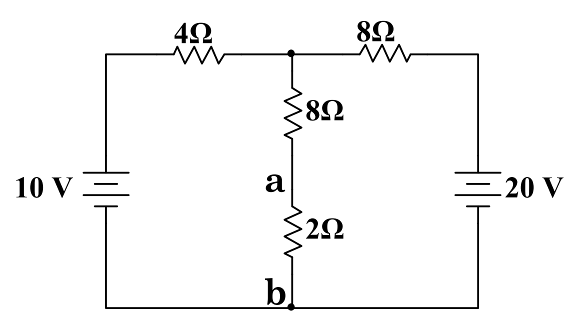


$I_n = 9 \times 3 / 6 = 4.5A.$

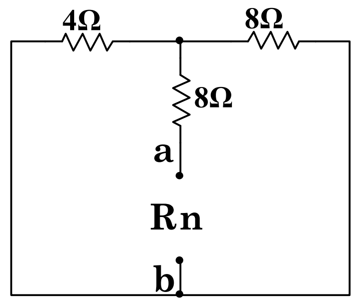
$R_n = 6 // 6 = 3\Omega$



Example : Calculat the potential difference across the 2Ω resistance in the network shown below .



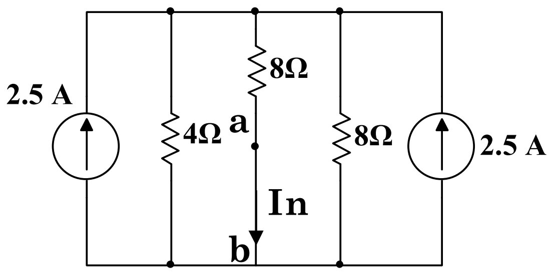
Solution : To find R_n
Remove 2Ω resistance

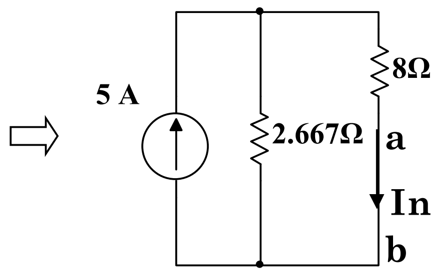


source 10V is short- circuited and source 20V is short- circuited.

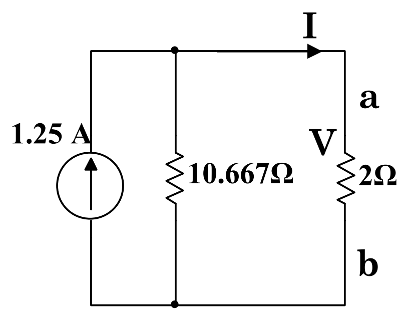
$R_n = 8 + (4//8) = 10.667\Omega$

By using conversion voltage source to current source





$I_n = 5 \times 2.667 / 10.667 = 1.25A.$



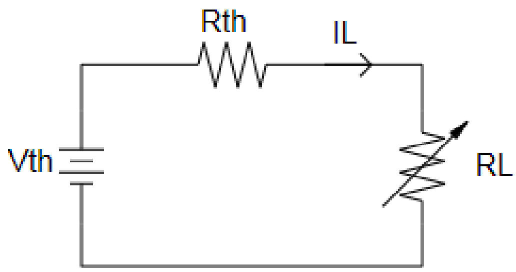
$I = 1.25 \times 12.667 / 12.667 = 1.053A$

$V = I \times 2 = 2.0154V.$

3-6 Maximum Power Transfer Theorem

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{th}$).

If the entire circuit is replaced by its Thevenin equivalent except for the load,as shown in Fig. below



$I_L = V_{th} \ / \ R_{th} + R_L$

the power delivered to the load is

$P = I_L^2 R_L = (V_{th} / R_{th} + R_L)^2 \ R_L = V_{th}^2 R_L \ / (R_{th} + R_L)^2$

To prove the maximum power transfer theorem, we differentiate P in Eq.over with respect to R_L and set the result equal to zero. We obtain

$$\frac{dp}{dR_L} = V_{\text{Th}}^2 \left[\frac{(R_{\text{Th}} + R_L)^2 - 2R_L(R_{\text{Th}} + R_L)}{(R_{\text{Th}} + R_L)^4} \right]$$
$$= V_{\text{Th}}^2 \left[\frac{(R_{\text{Th}} + R_L - 2R_L)}{(R_{\text{Th}} + R_L)^3} \right] = 0$$

This implies that

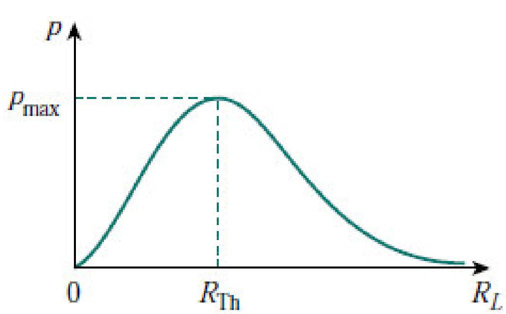
$$(R_{Th} + R_L - 2R_L) = 0$$

which yields

$$R_L = R_{th}$$

Thus

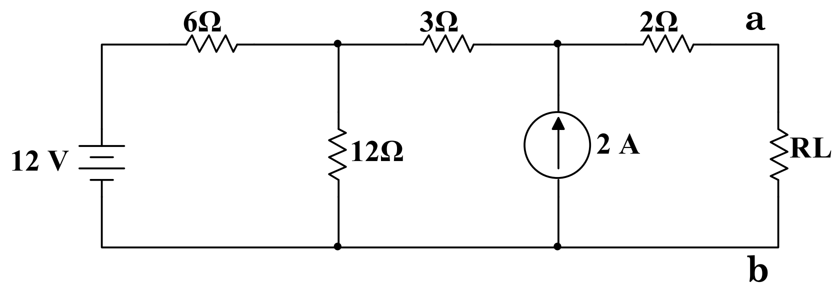
$P_{max} = I_L^2 R_L = V_{th}^2 R_L / (R_{th} + R_L)^2 = V_{th}^2 R_L / 4R_L^2 = \boxed{V_{th}^2 / 4R_L}$



at maximum power transfer condition:

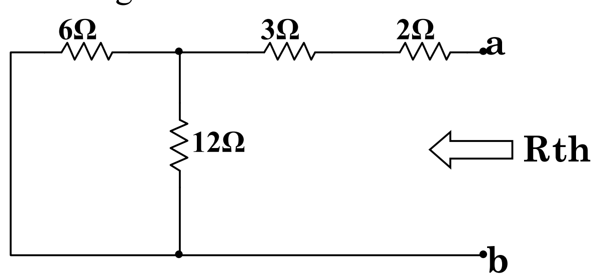
- $V_L = V_{th} / 2$
- $\eta = 50\%$

Example : Find the value of R_L for maximum power transfer in the circuit of Fig. shown below. Find the maximum power.



Solution:

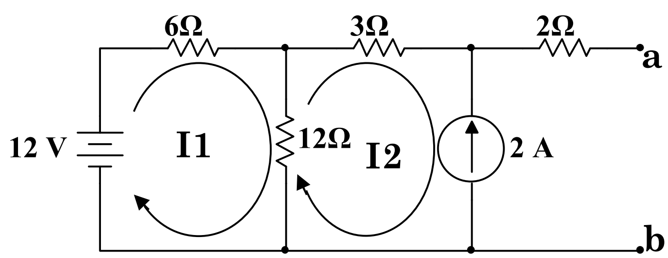
We need to find the Thevenin resistance RTh and the Thevenin voltage V_{th} across the terminals a-b. To get R_{th}, we use the circuit in Fig. over and obtain



$R_{th} = 2 + 3 + (6//12) = 9\Omega$

$R_L = R_{th} = 9\Omega$

To get V_{th}, we consider the circuit in Fig. .Applying maxwell's analysis



$18I_1 - 12I_2 = 12$, $I_2 = -2$ A

Solving for I_1,

we get $I_1 = -2/3$.

Applying KVL around the outer loopto get V_{th} across terminals a-b, we obtain

$6I_1 + 3I_2 + 2(0) + Vab = 12 \Rightarrow V_{ab} = 22 \text{ V} = V_{th}$

and the maximum power is

$P_{max} = V_{th}^2 / 4R_L = 22^2/4 \times 9 = 13.44$ W.

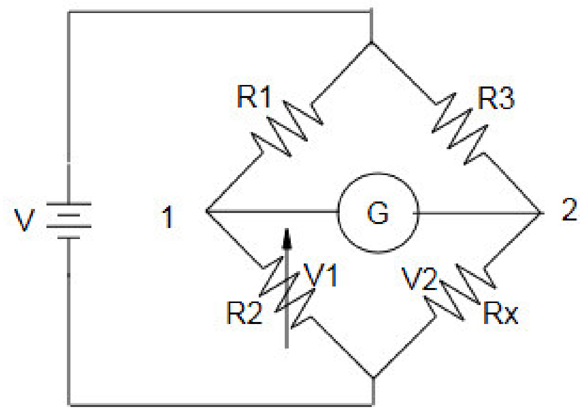
3-7 Wheatstonebridge

The Wheatstone bridge (or resistance bridge) circuit is used in a number of applica_ tions. Here we will use it to measure an unknown resistance.The unknown resistance R_x is connected to the bridge as shown in Fig. below. The variable resistance is adjusted until no current flows through the galvanometer(G) ,$I_G = 0$.

- When the current passing through the galvanometer equal zero $I_G = 0$

  $V_1 = V_2$

  $V_1 = V R_2 / R_1 + R_2$
  $V_2 = V R_X / R_X + R_3$



$V_1 = V_2 \Longrightarrow \quad V R_2 / R_1 + R_2 = V R_X / R_X + R_3$

$$Rx = R3 \times \frac{R2}{R1}$$

Chapter Four
Basic AC Theory

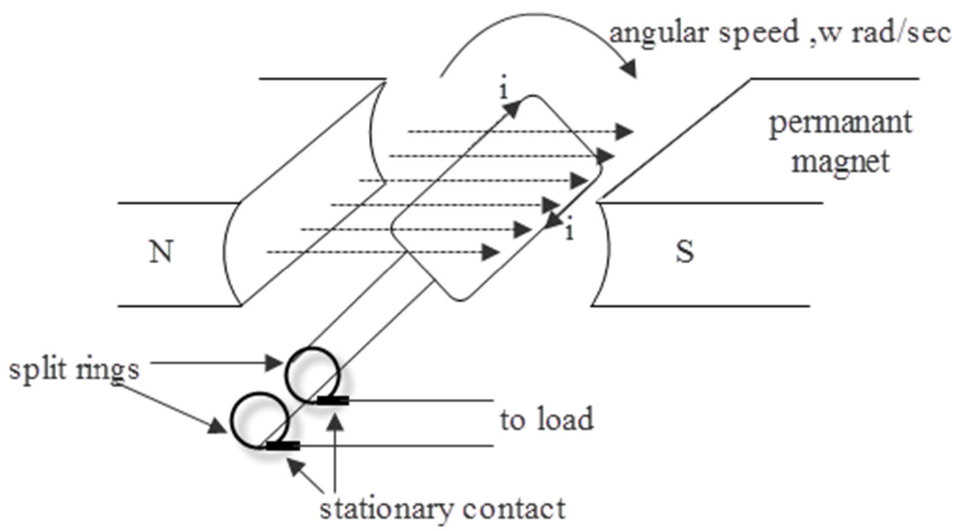
Introduction

Most students of electricity begin their study with what is known as direct current (DC), which is electricity flowing in a constant direction, and/or possessing a voltage with constant polarity. DC is the kind of electricity made by a battery (with definite positive and negative terminals).

An alternating quantity is one that regularly acts first in one direction and then in the opposite direction and do not have constant magnitude whith time .It magnitude continuously very whith time.

4-1Generation of AC Voltages

One way to generate an ac voltage is to rotate a coil of wire (conductor) at constant angular velocity in a uniform magnetic field ,an emf will be induced . The emf produced in this way is know as dynamically induced emf. As the fig.shown below



This results in the output from the generator being a sinusoidal (sinewave) voltage. The output voltage from an a.c. generator (alternator) is sinusoidal (a sinewave).

The voltage produced depends on the

1-Flux Density of the magnetic field produced by the magnet,

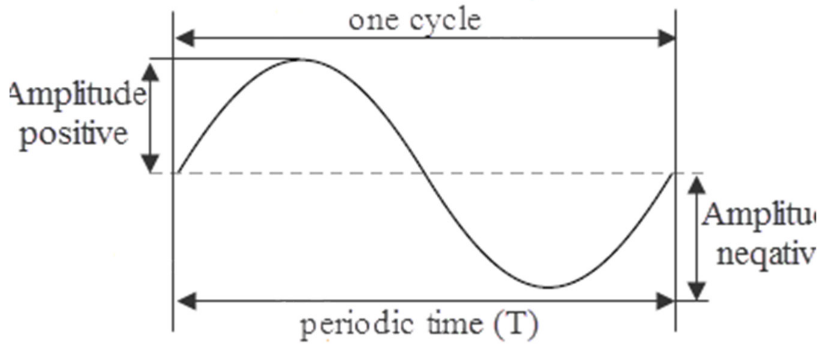
2- the number of turns on the two coils,

3-and the speed at which the rotor is moving.

-A sinusoid is a signal that has the form of the sine or cosine function.

4-2 AC Waveform Generated

An alternating waveform is a periodic waveform which alternate between positive and negative values and an alternating quantity charges whith time . It is also called waveshape.



- AC Waveform Characteristics

•Period (cycle) of a wave is one complete revolution of its shape until the point that it is ready to repeat itself.
• Periodic time (T) is the length of time in seconds that the waveform takes to repeat it self from start to finish. This can also be called the *Periodic Time* of the waveform for sine waves, or the *Pulse Width* for square waves.

•Frequency is the number of cycles that occur in one second of the alternating quantity . Electricity in iraq is produced at 50 cycles per second, the frequency of (50Hz) and this means one complete revolution of the alternator every20 milliseconds.

1 cycle /sec = 1 HZ

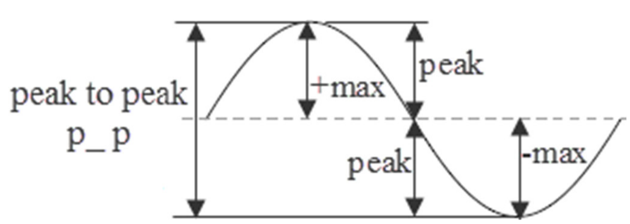
$f= 1/T$

T: Is the time of one cycle.

$f = nP$ hertz , P: the number of pole pairs , n: revolution per second (rev/s)
$f = 50\times1$ Hz (one pair of poles) = 50 Hz

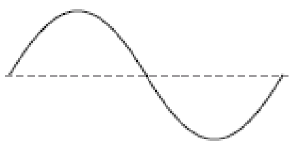
•Amplitude : The maximum value (positive or neqative) attained by alternating quantity . An amplitude measurement can take the form of peak, peak-to-peak, average, or RMS quantity.

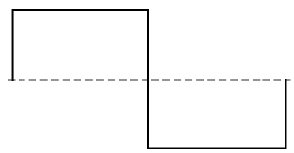
- Peak amplitude is the height of an AC waveform as measured from the zero mark to the highest positive or lowest negative point on a graph. Also known as the crest amplitude of a wave.
- Peak-to-peak amplitude is the total height of an AC waveform as measured from maximum positive to maximum negative peaks on a graph. Often abbreviated as "P-P".

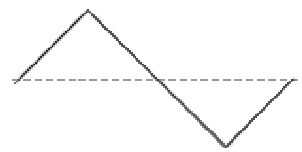


•Instantaneous value: Is the value of the waveform at any given instant of time. It is a time variable a(t).It is taken as υ(t) or e(t) for the voltage and i(t) for the current. Can be calculated by multiplying the peak value and the sine of the offset angle. For a sinusoid, Instantaneous value $a(t) = A_m \sin(\omega t)$.

4-3 A Typical Signal Generator
Can produce a variety of variable-frequency waveforms, including sinusoidal, square wave, triangular, and so on.


wave Sine


Square wave


Triangle wave

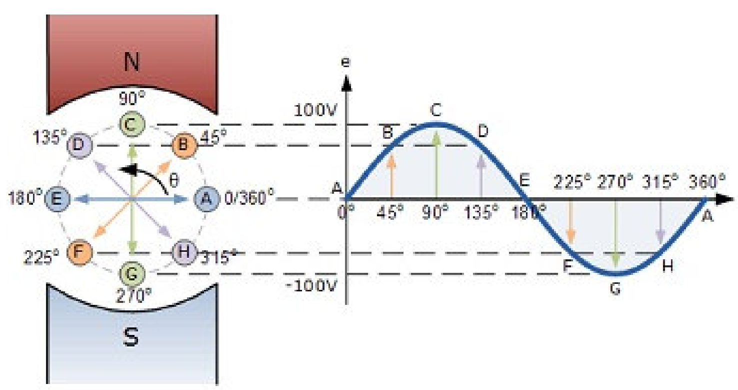
4- 4 Standard Expression for an Alternating Quantity
The generated waveform can be expressed algebraically

$e = E_m \sin\theta$ volt $E_m = \omega N \varphi_m$
or, $e = E_m \sin(\omega t)$ volt
or, $e = E_m \sin(2\pi f\, t)$ volt

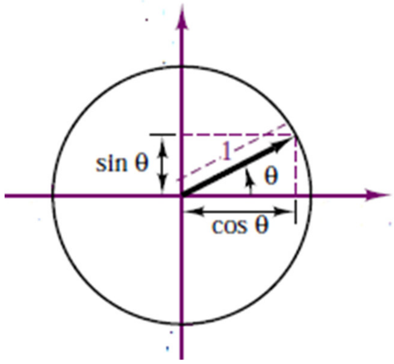
E_m : the maximum amplitude of the sinusoid
ω : the angular velocity in radians/s

$\theta = \omega t$
ω = angle / time = (θ / t) rad /sec
$\omega = 2\pi / T = 2\pi f$ rad / sec

4-5 Introduction to Phasors
A **phasor** is a rotating line whose projection on a vertical axis can be used to represent sinusoidally varying quantities. To get at the idea, observe the fig. shown below



Take point A as the starting point or zero phase. The phase at Point B is 45°, Point C is 90°, Point D is 135°, and so on, until Point A where the phase is 360°, or zero.

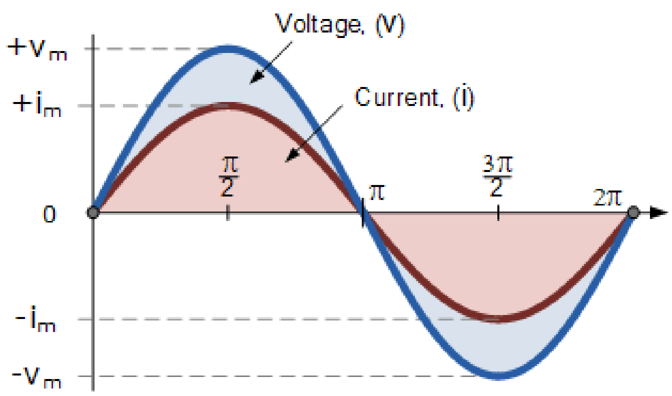


At any an angle taken will be there phasor diagram , as in the fig. shown

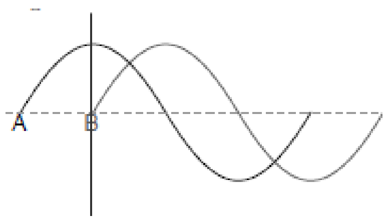
$$\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = r = 1$$

4- 6 Phase Difference or Phase Shift

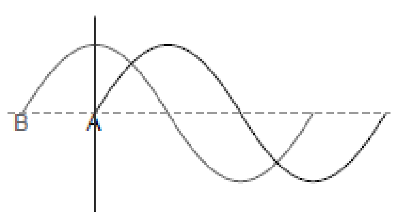
The **phase difference** or phase shift as it is also called of a Sinusoidal Waveform is the angle Φ (Greek letter Phi), in degrees or radians that the waveform has shifted from a certain reference point along the horizontal zero axis. In other words phase shift is the lateral difference between two or more waveforms along a common axis and sinusoidal waveforms of the same frequency can have a phase difference.The phase difference, Φ of an alternating waveform can vary from between 0 to its maximum time period, T .



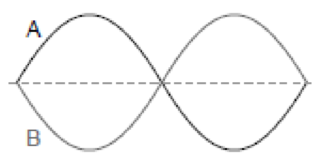
When two alternating quantities with the same frequency have different zero points (reference) they are said to have phase difference. In the feg. shown have zero phase difference .



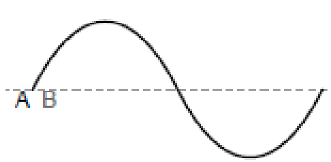
Phase shift = 90 degrees
B is ahead of A
(B "leads" A)



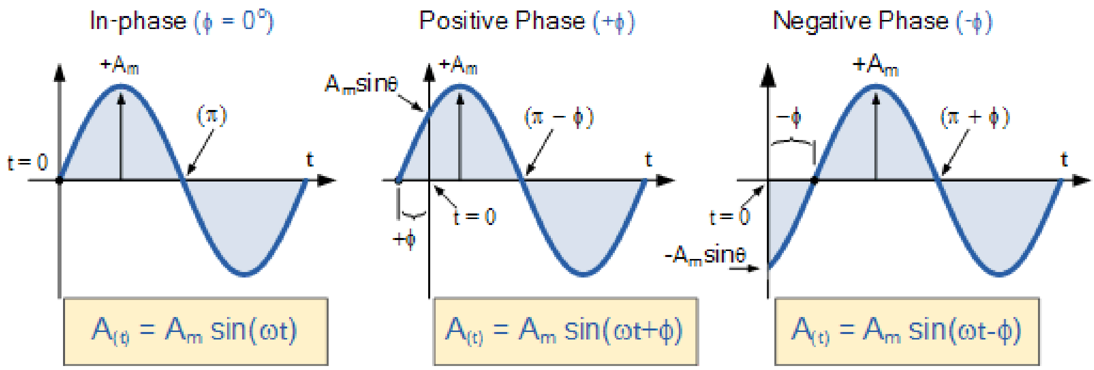
Phase shift = 90
A is ahead of B
(A "leads" B)



degrees Phase shift = 180 degrees
A and B waveforms are
mirror-images of each other



Phase shift = 0 degrees
A and B waveforms are
in perfect step with each other



$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin(\omega t \pm 90^\circ) = \pm\cos \omega t$

$\cos(\omega t \pm 90^\circ) = \mp\sin \omega t$

$\cos(\omega t) = \sin(\omega t + 90^\circ)$

$\sin(\omega t) = \cos(\omega t - 90^\circ)$

$-\cos(\omega t) = \cos(\omega t \pm 180^\circ)$

$-\sin(\omega t) = \sin(\omega t \pm 180^\circ)$

$-\cos(\omega t) = \sin(\omega t \pm 270^\circ)$

$-\sin(\omega t) = \sin(-\omega t)$

$\cos(\omega t) = \cos(-\omega t)$

4-7 Average value and Effective value of AC auantity

a- Average or Mean Value

The average value of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is tranaferred by that alternating current during the same time. Average values are also called **dc values**.

$I_{av} = (i_1 + i_2 + \ldots + i_n) / n$

$$\text{Average Value} = \frac{\text{area under curve}}{\text{length of base}}$$

-For unsymmetrical waveform

$$\text{Average Value} = \frac{\text{area under one cycle}}{\text{baselength of one cycle}}$$

-For symmetrical waveform

$$\text{Average Value} = \frac{\text{area under half cycle}}{\text{baselength of half cycle}}$$

For an ac waveform represented by function v(t) or i(t) the average value is :

$$V_{av} = \frac{1}{T}\int_0^T v(t)\,dt$$

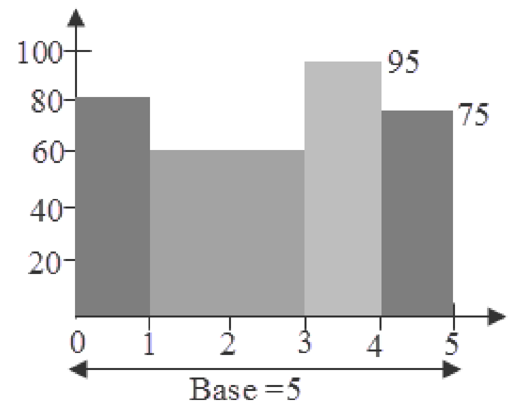
$$I_{av} = \frac{1}{T}\int_0^T i(t)\,dt$$

The mean value of a waveform which has equal positive and negative half cycles must thus be always zero. For this reason, the average value is taken to be the average over one half cycle.

Example : Determining average value for the fig.shown below.

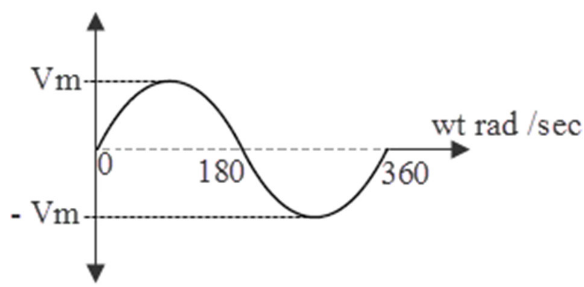
Solution:
Base =5

Area = (80×1)+(60×2)+(95×1)+(75×1)
= 370
Average = 370 / 5 = 74



Example : Find the average value of the sinusoidal voltage shown below.

Solution : The average value is taken to be the average over one half cycle.



$$V_{av} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} Vm\,Sin(wt)\,dt$$

$$= \frac{-Vm}{\pi}\,Coswt\,\Big|_0^{\pi} = \frac{-Vm}{\pi}[-1-1] = \frac{2\,Vm}{\pi} = 0.6366\,V_m \;,\; I_{av} = 0.6366 I_m$$

b- Effective Value or Root Mean Square (rms)
The r.m.s. value of an alternating current is equivalent to that value of direct current, which when passed through an identical circuit, will dissipate exactly the same amount of power. The r.m.s. value of an a.c. thus provides a means of making a comparison between a.c. and d.c. systems.

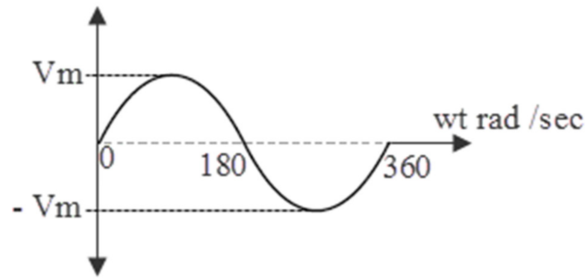
$$I_{r.m.s} = \sqrt{(\, i_1^2 + i_2^2 + i_n^2 \,)/n}$$

the root-mean-square, or rms, value of the alternating signal taken to full cycle ,and defined as follows:

r.m.s value = $\sqrt{\frac{1}{T}\int_0^T a^2(t)\,dt}$ in general where a(t) is current or voltage

Example :Find the effect value of the sinusoidal voltage shown below.

Solution :



$$V_{r.m.s} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(Vm\ Sin\ wt)^2\,dwt}$$

$$= \sqrt{\frac{(Vm)^2}{2\pi}\int_0^{2\pi}(Sin\ wt)^2\,dwt}$$

$$= \sqrt{\frac{(Vm)^2}{2\pi}\int_0^{2\pi}\frac{(1-Cos2wt)}{2}\,dwt} = \sqrt{\frac{(Vm)^2}{2\pi}\left(\frac{1}{2}\int_0^{2\pi}dwt - \frac{1}{2}\int_0^{2\pi}Cos2wt\,dwt\right)}$$

$$= \sqrt{\frac{Vm^2}{2\pi}\times\frac{2\pi}{2}} = \frac{Vm}{\sqrt{2}} = 0.707\ V_m \quad ,\ I_{r.m.s} = 0.707\ I_m$$

4-8 Form Factor and Crest Factor(peak factor)

Although little used these days, both **Form Factor** and **Crest Factor** can be used to give information about the actual shape of the AC waveform.

- Form Factor

As the name implies, this factor gives an indication of the form or shape of the waveform. It is defined as the ratio of the r.m.s. value to the average value. Thus, *for a sinewave*

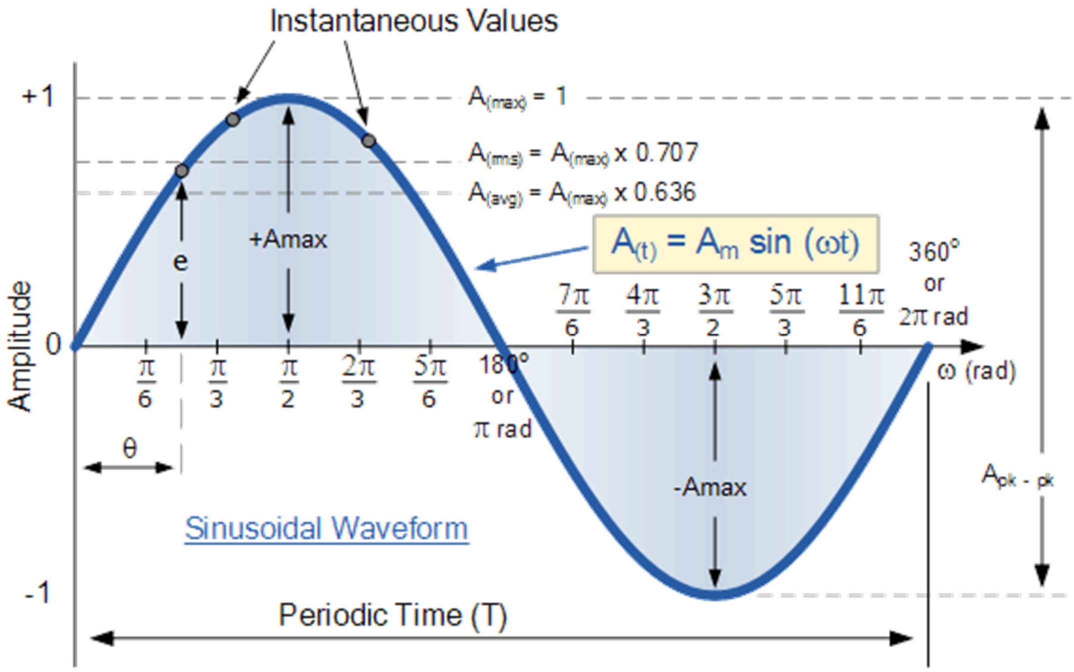
$$\text{form factor } (k_f) = \frac{\text{r.m.s.value}}{\text{average value}} = \frac{0.707Vm}{0.637Vm}$$

so, form f actor =1.11

- Peak Factor

This is defined as the ratio of the peak or maximum value, to the r.m.s. value, of a waveform. Thus, *for a sinewave only*

$$\text{peak factor } (k_a) = \frac{\text{maximum value}}{\text{r.m.s.value}} = \frac{Vm}{0.707\,Vm} = \sqrt{2} \text{ or } 1\ 414$$



quantity	equations	expression
$V_{r.m.s}$	$V_m/\sqrt{2}$	0.707Vm
$I_{r.m.s}$	$I_m/\sqrt{2}$	0.707 Im
V_{av}	$2V_m/\pi$	0.637Vm
I_{av}	$2I_m\ /\pi$	0.637 Im
K_f	r.m.s./av.	1.11
K_a	max. / r.m.s.	1.414 or $\sqrt{2}$

Examples

Ex.1 : The current in a circuit is given by: $i(t) = 141.4\sin(377t)$A.

find the values of :

- the rms current ,

-the frequency ,

-and the instantaneous value of the current when t=3ms.

Solution:

$i=I_m \sin wt = 141.4 \sin 377t$

- $I_{r.m.s.} = I_m/\sqrt{2} = 0.707\times 141.4 = 100$A.

- $w = 377$

$w = 2\pi f \implies f = w/2\pi = 377/2\pi = 60$HZ.

- At t=3ms

$i = 141.4\sin(377\times 3\times 10^{-3}) = 141.4\sin(1.131)$, $1.131\times 180/\pi = 64.9^{o}$

$i = 141.4\sin 64.9 = 128.9$A.

Ex.2: An alternating voltage is represented by the expression $v = 35\sin(314.2\,t)$ volt. Determine, (a) the maximum value, (b) the frequency, (c) the period of the waveform, and (d) the value 3.5 ms after it passes through zero, going positive.

Solution: $v = 35\sin(314.2t)$ volt and comparing this to the standard,

a) $v=V_m\sin(2\pi f\,t)$ volt we can see that: $V_m = 35v$

b) Again, comparing the two expressions:

$2\pi f=314.2 \Longrightarrow f = 314.2/\,2\pi = 50HZ.$

c) $T= 1\,/\,f =1/50$ second

so, $T=20ms$ **Ans**

(d) When t=3.5ms; then:

$v=35\sin(314.2\times 3.5\times10^{-3}) = 35\sin(1.099)^{*} = 35\sin(62.97) = 35\;0.891= 31.19v.$

*The term inside the brackets is an angle in Radian.Must transfe it to degree.

Ex.3: For a current, $i = 75\sin(200\,\pi\,t)$ mA, determine
(a) the frequency, and (b) the time taken for it to reach 35 mA, for the first,after passing through zero.

Solution:

$i= 75\sin(200\,\pi\,t)$ mA = $Im\sin(wt)$ mA

a) $w = 2\pi f \Longrightarrow 200\pi = 2\pi f \Longrightarrow f = 200\pi\,/\,2\pi = 100HZ.$

b) $35\text{ mA} = 75\sin(200\pi t)$

$35/75 = \sin(200\pi\,t) \Longrightarrow 0.4667 = \sin(200\pi t) \Longrightarrow 200\pi t = \sin^{-1} 0.4667 = 0.4855rad$

$200\pi t = 0.4855 \Longrightarrow t = 0.4855\,/\,200\pi = 0.773$ ms.

Ex.4: A sinusoidal alternating voltage has an average value of 3.5 V and a period of 6.67 ms. Write down the standard (trigonometrical) expression for this voltage.

Solution: $V_{av} = 3.5$ V ; $T = 6.67\times10^{-3}$ s

The standard expression is of the form $v = V_m\sin(2\pi f\,t)$ volt

$V_{av} = 0.637\,V_m \Longrightarrow V_m = V_{av}\,/\,0.637 = 3.5\,/\,0.637 = 5.5V$

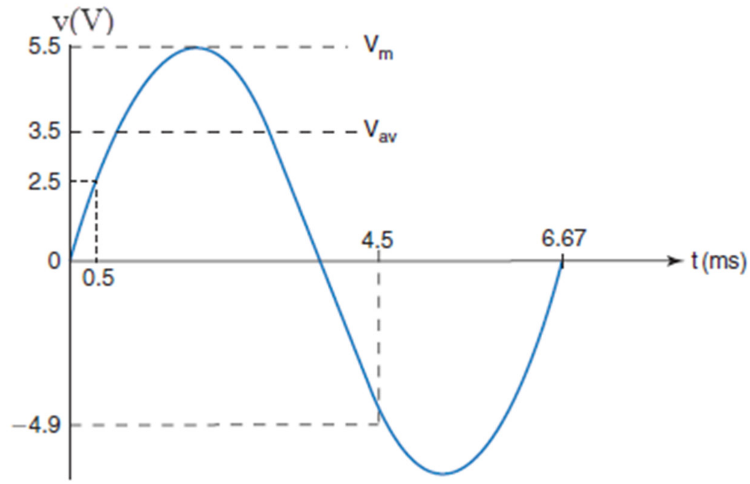
$f = 1/\,T = 1/(\,6.67\times 10^{-3}) = 150HZ.$

$v = 5.5\sin(2\pi150t)V = 5.5\sin(300\pi t)V$.

Ex.5: For the voltage $v= 5.5\sin(300\pi t)V$, after the waveform passes through zero, going positive, determine its instantaneous value (a) 0.5 ms later, (b) 4.5 ms later, and (c) the time taken for the voltage to reach 3 V for the first time.

Solution : (a) $t = 0.5 \times 10^{-3}$ s ; (b) $t = 4.5 \times 10^{-3}$ s ; (c) v = 3 V

a) $v = 5.5 \sin(300\pi \times 0.5 \times 10^{-3})V = 5.5\sin 0.4712 = 5.5 \sin 27^o = 5.5 \times 0.454 = 2.5V$.
b) $v = 5.5 \sin(300\pi \times 4.5 \times 10^{-3})V = 5.5\sin 4.241 = 5.5 \sin 243^o = 5.5 \times (-0.891) = -4.9V$.
c) $3 = 5.5 \sin(300\pi t)V$
$3/5.5 = \sin(300\pi t) \Longrightarrow 0.5455 = \sin(300\pi t) \Longrightarrow 300\pi t = \sin^{-1} 0.5455$
$300\pi t = 33.058\pi / 180 = 0.5769$ rad
$t = 0.5769 / 300\pi = 0.612$ms.



Ex.6: Calculate the amplitude of the household 240 V supply.
Solution :
$V_{rms} = V_m / \sqrt{2} \Longrightarrow V_m = 240 \times \sqrt{2} = 339.4v$.

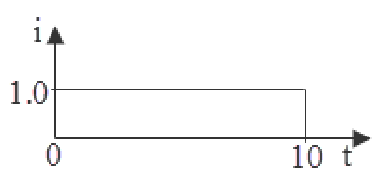
Ex.6: Three alternating currents are specified below. Determine the frequency,and for each current, determine its phase angle, and amplitude.

$i_1 = 5 \sin(80\pi t + \pi/6)$ amp
$i_2 = 3 \sin(80\pi t)$ amp
$i_3 = 6 \sin(80\pi t - \pi/4)$ amp

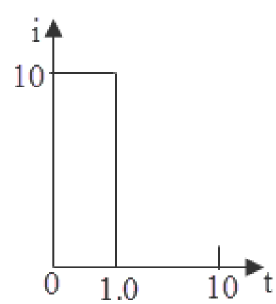
Solution : All three waveforms have the same value of w , namely 80 π rad/s. Thus all three have the same frequency:
$w = 2\pi f = 80\pi \Longrightarrow f = 80\pi / 2\pi = 40HZ$.

$I_{m2} = 3V$ and $\emptyset_2 = $ zero
$I_{m1} = 5V$ and leads i_2 by $\emptyset_1 = \pi/6$ rad (30^o)
$I_{m3} = 6V$ and lags i_2 by $\emptyset_3 = \pi/4$ rad (45^o)

Example:Calculate the form factor for the waveform shown in below.



a



b

Solution :

a) $I_{av} = (1\times10)/10 = 1A$

$I_{rms} = ((1\times10)^2/10)^{1/2} = 3.16A$

$K_f = r.m.s./av. = 3.16A$

b) $I_{av} = (10\times1)/10 = 1A$

$I_{rms} = ((10\times1)^2/10))^{1/2} = 3.16A$

$K_f = r.m.s./av. = 3.16A$

4-8 Simple Single Element Circuit

1- Resistance

$v(t) = V_m \sin wt$

$i(t) = v(t) / R$

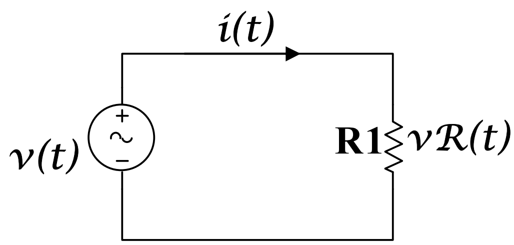
$= V_m \sin wt / R$

- $i(t) = I_m \sin wt$

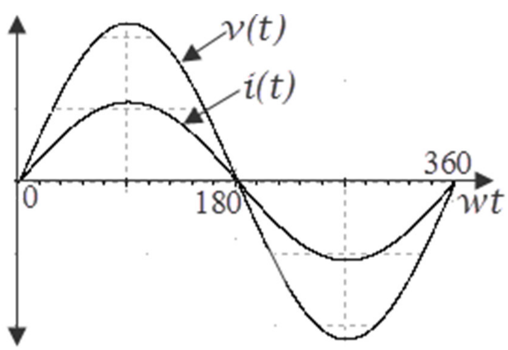
where

$I_m = V_m / R$

$i(t) = i_R(t)$



The phase difference between the two signal is zero, means that phase angle $\emptyset = 0$ as in the waveform shown below



Phasor diagram

2- Inductance

$v(t) = V_m \sin wt$

$i(t) = v(t) / X_L$

$= V_m \sin wt / X_L$

$= I_m \sin wt$

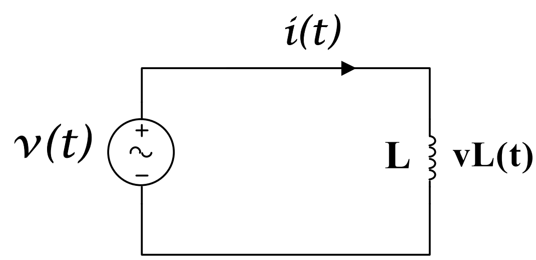
$v_L(t) = L\ (di / dt)$

$= L\ (d\ i(t) / dt) = L\ (d(I_m \sin wt) / dt)$

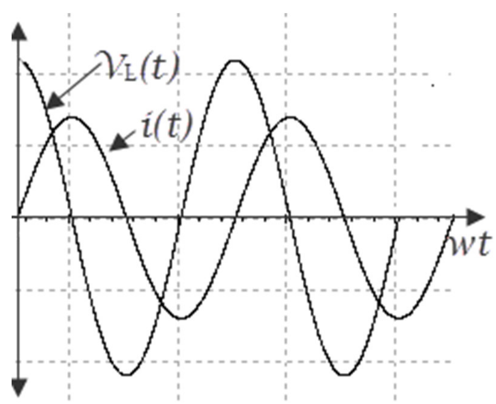
$= wL\ I_m \cos wt = V_m \cos wt = V_m \sin(wt + \frac{\pi}{2})$

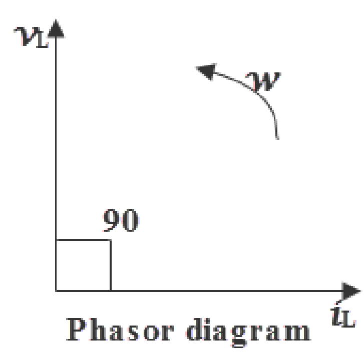
$V_m = wL\ I_m$; $V_m = X_L\ I_m$

$X_L = wL$

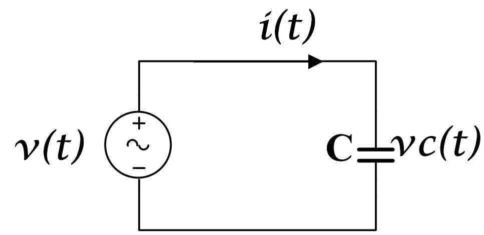


The current in a purely i inductive circuit lags the voltage vector by 90º. As in the forms below





3- Capacitance



$v(t) = V_m \sin wt = v_c(t)$

$i(t) = C\ (dv_c(t) / dt)$

$= C\ (d(V_m \sin wt\ )/dt)$

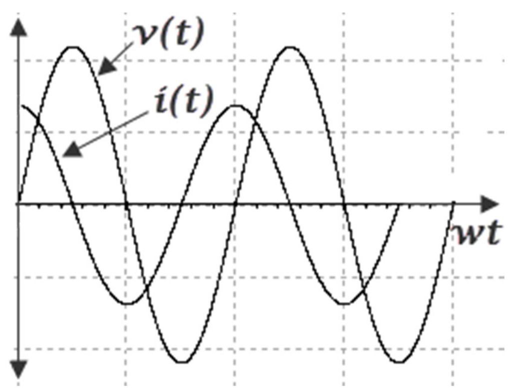
$= wC\ V_m \cos wt\ = V_m \cos wt\ / Xc$

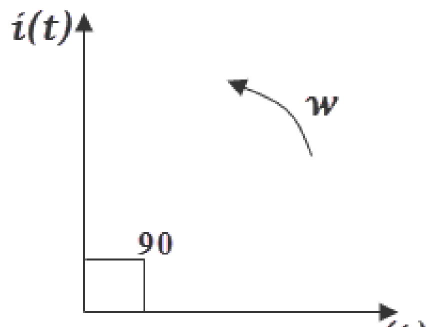
$= I_m \cos wt = I_m \sin(wt + \frac{\pi}{2})$

Where

$wc = 1/\ X_C$, $I_m = V_m / X_C$

The current leads the voltage by 90^o in acapacitor.As in forms below



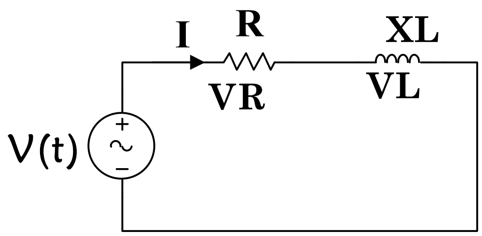


Phasor diagram

Chapter Five
Analysis of Single Phase AC Circuits

5-1 Series AC Circuits

1- Series R- L Circuit



$V_R = I\,R$

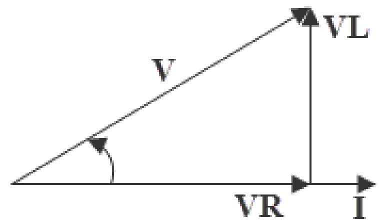
$V_L = I\,X_L$

- $V = V_R + j\,V_L$ (rectangular form)

  $V = |V|\angle\theta$ (polar form)

  $|V| = \sqrt{{V_R}^2 + {V_L}^2}$ polar magnitude

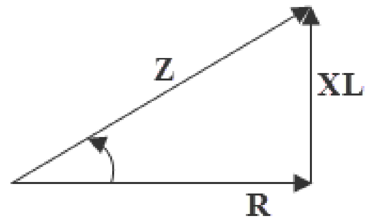
  $= I\,|Z|$



- $Z = R + j\,X_L$ (rectangular form)

  $Z = |Z|\angle\varphi$ (polar form)

  $|Z| = \sqrt{R^2 + {X_L}^2}$ polar magnitude

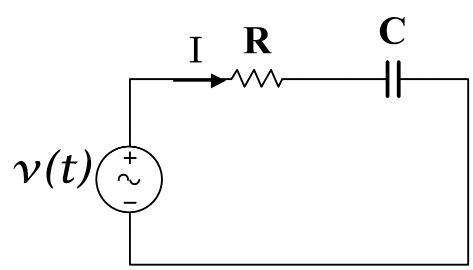


$R = Z\cos\varphi$

$X_L = Z\sin\varphi$

$$\tan\varphi = \frac{V_L}{V_R} = \frac{I\,X_L}{I\,R} = \frac{X_L}{R}$$

2- Series R- C Circuit



$V_R = I\,R$

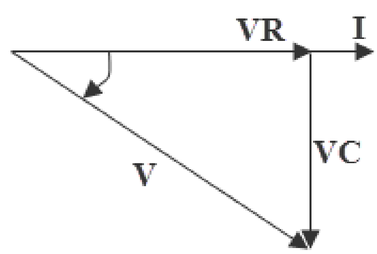
$V_C = I\,X_C$

- $V = V_R - j\,V_C$ (rectangular form)

  $V = |V|\angle\theta$ (polar form)

  $|V| = \sqrt{{V_R}^2 + {V_C}^2}$ polar magnitude

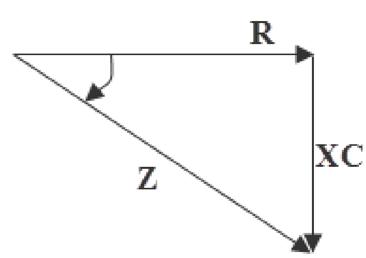
  $= I\,|Z|$



- $Z = R - j\,X_C$ (rectangular form)

  $Z = |Z|\angle -\varphi$ (polar form)

  $|Z| = \sqrt{R^2 + {X_C}^2}$ polar magnitude



$R = Z\cos\varphi$

$X_C = Z\sin\varphi$

$$\tan\varphi = \frac{-V_C}{V_R} = \frac{-I\,X_C}{I\,R} = \frac{-X_C}{R}$$

Z : impedance

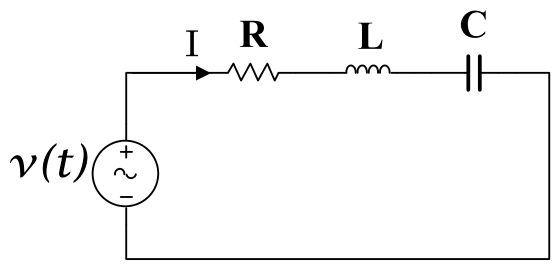
X : reactance

X_L : inductive reactance

X_C : capacitive reactance

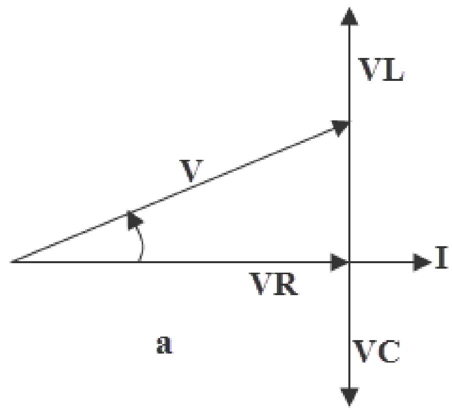
3- R-L-C Series Circuit

Impedance in an R-C-L series circuit is equal to the phasor sum of resistance, inductive reactance, and capacitive reactance



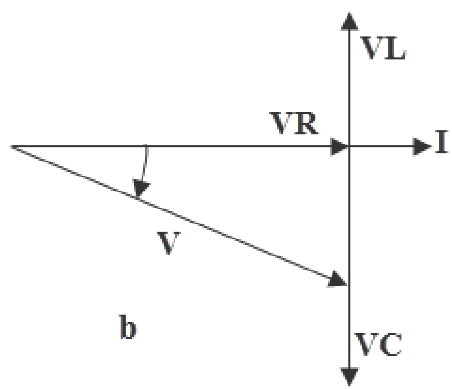
- $V = V_R + j(V_L + V_C)$
  $= |V|\angle\theta$

$$|V| = \sqrt{V_R^{\,2} + (V_L - V_C)^2}$$
$$= I\,|Z|$$
$$|Z| = \sqrt{R^2 + X^2}$$
$$X = X_L - X_C$$
$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{I\,(X_L - X_C)}{I\,R} = \frac{X_L - X_C}{R}$$
$$= X / R$$



•There are three probabilities

1- the reactance ($X = X_L - X_C$) is positive when $X_L > X_C$,for inductive circuit and (φ) is positive as in fig.a for lagging power factor.

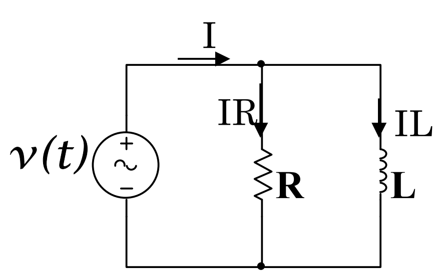


2- the reactance ($X = X_L - X_C$) is negative when $X_C > X_L$,for capacitive circuit and (φ) is negative as in fig.b for leading power factor.

3- the reactance ($X = X_L - X_C$) equal zero when $X_L = X_C$,for resistive circuit and (φ) equal zero.

5- 2 Parallel AC Circuits

1- Parallel R- L Circuit



$I = I_R + I_L$

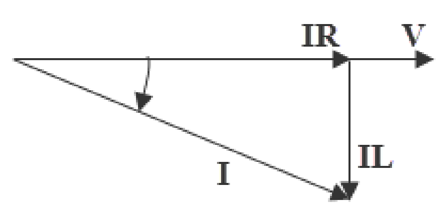
$V/Z = V/R - j\,V/X_L = V\,(1/R - j\,/\,X_L)$

$I = V\,Y$

$Y = 1/Z = 1/R - j/X_L = G - jB$

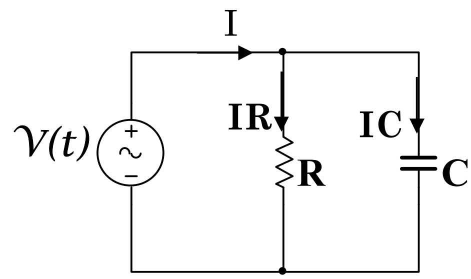
$I = |I|\angle\varphi$

$$|I| = \sqrt{I_R^{\,2} + I_L^{\,2}} \quad , \tan\varphi = \frac{I_L}{I_R} = \frac{V/X_L}{V/R} = \frac{R}{X_L}$$



Y : admittance
G : conductance
B : suceptance

2- Parallel R- C Circuit



$I = I_R + I_C$

$V/Z = V/R + j\,V/X_C = V\,(1/R + j\,/\,X_C)$

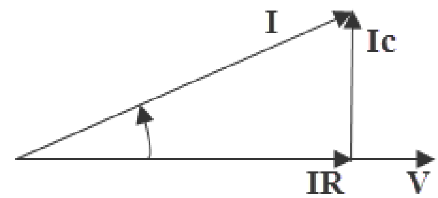
$I = V\,Y$

$Y = 1/Z = 1/R + j/X_C = G + jB$

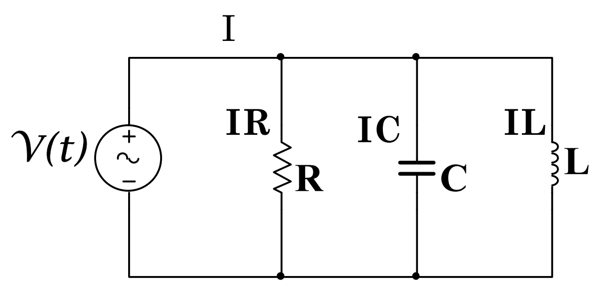
$I = |I|\angle\varphi$

$|I| = \sqrt{I_R^{\,2} + I_C^{\,2}}$

$\tan\varphi = \frac{I_C}{I_R} = \frac{V/X_C}{V/R} = \frac{R}{X_C}$



3- R-L- C Parallel Circuit



$I = I_R + jI_C - j\,I_L = I_R + j(I_C - I_L)$

$= V/Z = V\,Y$

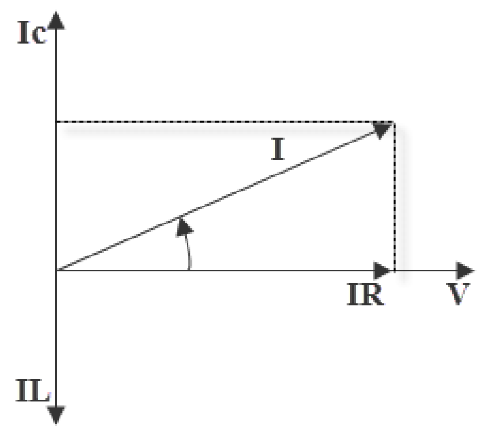
$Y = \frac{1}{Z} = \frac{1}{R} + j\left(\frac{1}{XC} - X\,L\right)$

$= G + j(\,B_C - B_L\,)$

$\tan\varphi = \frac{I_C - I_L}{I_R} = \frac{B_C - B_L}{G}$

•The formulas for finding total current (I_T) in a parallel R-C-L circuit are:

1- where $I_C > I_L$, $I_X = I_C - I_L$, for capacitive circuit and leading power factor.
2- where $I_L > I_C$, $I_X = I_L - I_C$, for inductive circuit and lagging power factor.



5-3 Power in AC Circuit

The power cannot be calculated in AC circuits in the same manner as in DC circuits. The power triangle, shownin Figure below a way to represent the relationship between real or active power (P) in watts(w), reactive power (Q) in volt-amperes-reactive (VAR), and apparent power (S) in volt-amperes (VA) using a triangle.

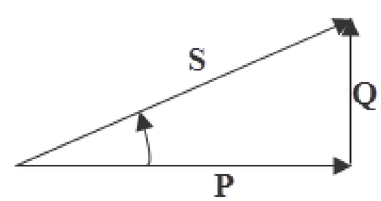
In general

$S = V\,I^* = |I|^2 Z = |V|/Z^* = P \pm jQ = |S|\angle \pm \varphi$, $|S| = \sqrt{P^2 + Q^2}$

$P = I^2 R = S\,\cos\varphi$ (in any circuit)

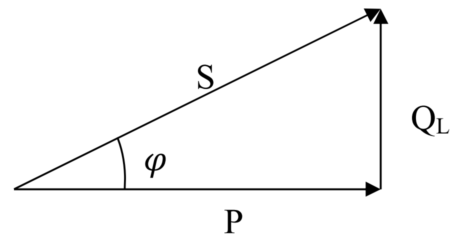
$Q = I^2 X = V^2/\,X = S\,\sin\varphi$

$\tan\varphi = \frac{Q}{P}$; (I & V in r.m.s value)



a-Inductive Circuit

In an inductive circuit, the current lags the voltage and is said to have a lagging power factor, as shown in Figure a.



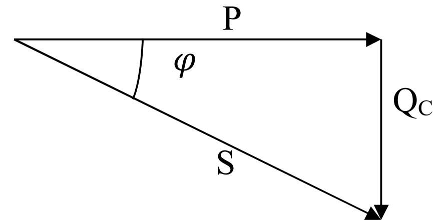
$S = V I^* = |I|^2 Z = P + jQ_L$

$Q_L = I^2 X_L = S \sin\varphi$

$\tan\varphi = \frac{Q_L}{P}$

b- Capacitive Circuit

In a capacitive circuit, the current leads the voltage and is said to have a leading power factor, as shown in Figure b.



$S = V I^* = |I|^2 Z = P - jQ_C$

$Q_C = I^2 X_C = S \sin\varphi$

$\tan\varphi = \frac{-Q_C}{P}$

Notes :

VARs are generated by capacitors and absorbed by inductors .The phase, φ, of the absorbed power, S, equals the phase of Z.

If Q is negative, the circuit is capacitive; if positive, the circuit is Inductive.

Exampe : If $i_1 = 14.14 \sin(\omega t - 55°)$ A and $i_2 = 4 \sin(\omega t + 15°)$ A ,determine their sum i.Work with rms values.

Solution:

$I_1 = (0.707)(14.14\ A)\angle{-55°} = 10\ A\angle{-55°}$

$I_2 = (0.707)(4\ A)\angle 15° = 2.828\ A\angle 15°$

$I = I_1 + I_2 = 10\ A\angle{-55°} + 2.828\ A\angle 15°$

$= (5.74\ A - j8.19\ A) + (2.73\ A + j0.732\ A)$

$= 8.47\ A - j7.46\ A = 11.3\ A\angle{-41.4°}$

$i(t) = \sqrt{2}\,(11.3) \sin(\omega t - 41.4°) = 16 \sin(\omega t - 41.4°)$ A

Example : The voltage across a 0.2-H inductance is $v_L = 100 \sin(400t + 70°)$ V. Determine i_L .

Solution

$\omega = 400$ rad/s.

Therefore, $X_L = \omega L = (400)(0.2) = 80\ \Omega$.

$I_m = V_m / X_L = 100/80 = 1.25$ A

The current lags the voltage by 90°. Therefore $i_L = 1.25 \sin(400t - 20°)$ A .

By using the impedance concept.

$V_L = V_m / \sqrt{2} = 100 / \sqrt{2} = 70.7$ V∠ 70° and ω = 400 rad/s
$Z_L = j\,\omega L = j(400)(0.2) = j80\ \Omega$
$I_L = V_L / Z_L$ =70.7□ 70°/ j80 = 70.7□70 °/ 80□90° = 0.884 □-20° A.
In the time domain, $i_L = \sqrt{2}\,(0.884)\sin(400t - 20°) = 1.25\sin(400t - 20°)$ A.

Example : The current through a 0.01-H inductance is $i_L = 20\sin(\omega t - 50°)$ A and f = 60 Hz. Determine v_L.

Solution:
$\omega = 2\pi f = 2\pi(60) = 377$ rad/s
$X_L = \omega L = (377)(0.01) = 3.77\ \Omega$
$V_m = I_m X_L = (20\text{ A})(3.77\ \Omega) = 75.4$ V
Voltage leads current by 90°. Thus, $v_L = 75.4\sin(377t + 40°)$ V.

Example : The voltage across a 10-uF capacitance is $v_C = 100\sin(\omega t - 40°)$ V and f = 1000 Hz. Determine i_C.

Solution:
$\omega = 2\pi f = 2\pi(1000\text{ Hz}) = 6283$ rad/s
$X_C = 1/\omega C = 1/(6283)(10 \times 10^{-6}) = 15.92\ \Omega$
$I_m = V_m / X_C = 100 / 15.92V = 6.28$ A
Since current leads voltage by 90°, $i_C = 6.28\sin(6283t + 50°)$ A

By using the impedance concept.

$\omega = 2\pi f = 2\pi(1000\text{ Hz}) = 6283$ rads/s
$V_C = V_m / \sqrt{2} = 10 / \sqrt{2} = 70.7\ V\angle -40°$
$Z_C = 1/(j\omega C) = -j/\omega C = j/(628310 \times 10^{-6}) = -j15.92\ \Omega$
$I_C = V_C / Z_C = 70.7\angle -40° / -j15.92 = 70.7\angle -40° / 15.92\ \angle -90 = 4.442\ \angle 50°$ A
In the time domain, $i_C = \sqrt{2}\,(4.442)\sin(6283t + 50°) = 6.28\sin(6283t + 50°)$ A.

Example : Determine the effective values of
a. $i = 10\sin\omega t$ A
b. $i = 50\sin(\omega t + 20°)$ mA
c. $v = 100\cos 2\omega t$ V

Solution Since effective values depend only on magnitude,
a. $I_{eff} = 10\text{ A} / \sqrt{2} = 7.07$ A.
b. $I_{eff} = 50\text{ mA} / \sqrt{2} = 35.35$ mA.
c. $V_{eff} = 100\text{ V} / \sqrt{2} = 70.7$ V.

Example : Given e_1 = 10 sin ω*t* V and e_2 = 15 sin(ω*t* + 60°) V as before, determine *v* total.

Solution :

e_1 = 10 sin ω*t* V. Thus, **E**1 = 10 V∠ 0°.

e_2 = 15 sin(ω*t* + 60°) V. Thus, **E**2 = 15 V∠ 60°.

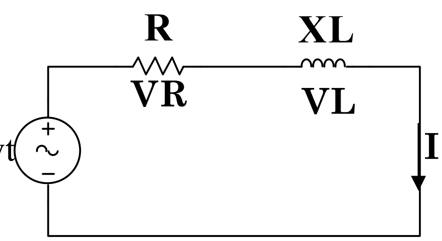
V = **E**1 + **E**2 = 10∠ 0° + 15∠ 60° = (10 +j0) + (7.5 + j13)

= (17.5 + j13) = 21.8 □ 36.6° V

Thus, *v* = 21.8 sin(ω*t* + 36.6°) V

Example : For the circuit shown in Figure below find Z, I,V_R ,and V_L , when R= 3Ω X_L = 4Ω .

Solution :



e = 141.4 sin ωt ⇒ **E** =100 ∠ 0° V

Z = R + jX_L = 3 + j4 = 5∠53.13° Ω

I = E / Z = 100 / 5∠53.13°

= 20∠-53.13° A

V_R = I R = (20∠-53.13°) × 3

= 60∠-53.13° V

V_L = I X_L = (20∠-53.13°)(4∠90°) = 80∠36.87° V

E = V_R + V_L = (60∠-53.13°) + (80∠36.87°) =(36 - j 48) + (64 + j 48) = 100 ∠ 0° V.

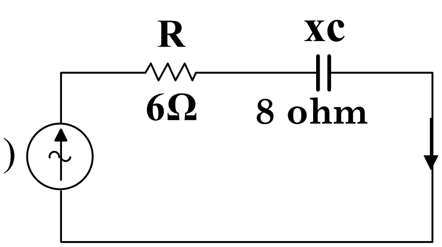
P.F = Cosφ = Cos53.13 = 0.6 lagging

P.F = Cosφ = R / Z = 3/ 5 = 0.6 lagging.

$\varphi = \tan^{-1}\frac{4}{3} =$ 53.1

Example : For the circuit shown in Figure below find Z, I,V_R ,and V_C .

Solution :



I = (7.07/$\sqrt{2}$) ∠53.13° A

= 5∠53.13° A

Z = R+ (1/ j2πf C) = R – j X_C

= 6 – j8 = 10 ∠-53.13°Ω

V_R = I R = (5∠53.13°) 6 = 18 + j24 = 30∠53.13° V

V_C = I X_C = (5∠53.13°)(8∠-90°) =32 –j 24 = 40 ∠-36.87° V

E = V_R + V_C = (18 + j24) +(32 –j 24) = 50 V.

E = I Z = (5∠53.13° A)(10 ∠-53.13°Ω) = 50 V.

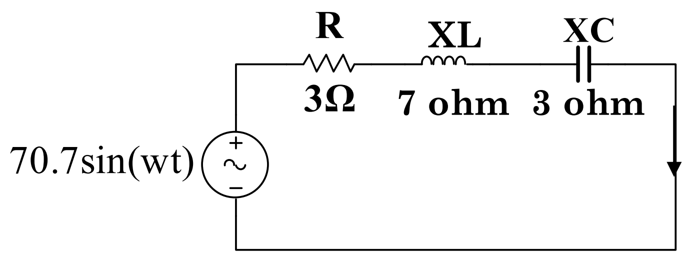
e = 50 $\sqrt{2}$ sin ωt = 70.70 sin ωt

P.F = Cosφ = Cos53.13 = 0.6 leading

P.F = Cosφ = R / Z = 6/ 10 = 0.6 leading.

$\varphi = \tan^{-1}\frac{-8}{6} =$ - 53.1

Example : For the circuit shown in Figure below find Z, I, V_R, V_L, and V_C.



Solution :

$E = (70.7/\sqrt{2}) \angle 0^o V = 50\angle 0^o V$

$Z = R + j X = R + j(X_L - X_C) = 3+ j(7 - 3) = 3+j4 = 5\angle 53.13^o \Omega$ (inductive circuit)

$I = E/Z = (50\angle 0^o) / (5\angle 53.13^o) = 10\angle - 53.13^o A$

$V_R = I R =(10\angle - 53.13^o) 3 = 30 \angle - 53.13^o V$

$V_L = I X_L = (10\angle - 53.13^o) (7\angle 90^o) = 70 \angle 36.87^o V$

$V_C = I X_C = (10\angle - 53.13^o) (3\angle -90^o) = 30 \angle -143.13° V$

$E = V_R + V_L + V_C = 50\angle 0^o$ V.

P.F = $\text{Cos}\varphi$ = Cos53.13 = 0.6 lagging

P.F = $\text{Cos}\varphi$ = R / Z = 3/ 5 = 0.6 lagging.

$\varphi = \tan^{-1}\frac{4}{3} = 53.1$

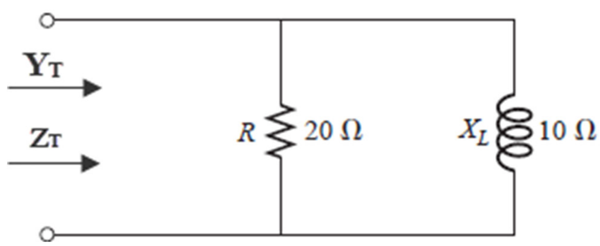
Example : For the network of Fig.below.

a. Find the admittance of each parallel branch.

b. Determine the input admittance.

c. Calculate the input admittance

d. Calculate the input impedance.



Solution :

a. $Y_R = G\angle 0^o =1/ R = 1/20\angle 0^o$

$= 0.05\angle 0^o = 0.05$ ℧ or S

b. $Y_L = B\angle 90^o = 1/ X_C = 1/ 10\angle 90^o$

$= 0.1\angle -90^o = - j0.1$ ℧ or S

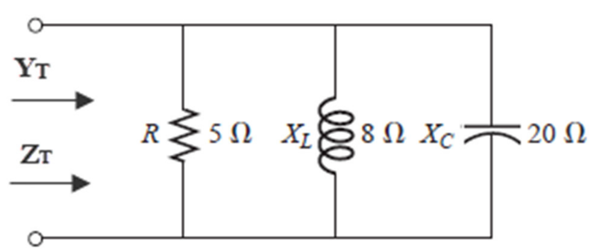
c. $Y_T = G + j B_L = Y_R + Y_L = 0.05 - j 0.1=0.1118\angle -63.435^o$ ℧ or S

d. $Z = 1/ Y_T = 1 / (0.05 - j 0.1) = 8.93 \angle -63.43° \Omega$.

or

$Z = (R X_L) / (R + X_L) = 8.93 \angle -63.43° \Omega$.

Example : Repeat Example above for the parallel network of fig.shown below.



Solution :

$Y_R = G = 1/5 = 0.2$ S

$Y_L = B_L = 1/ j8 = -j0.125$ S

$Y_C = B_C = 1/-j20 = j0.05$ S

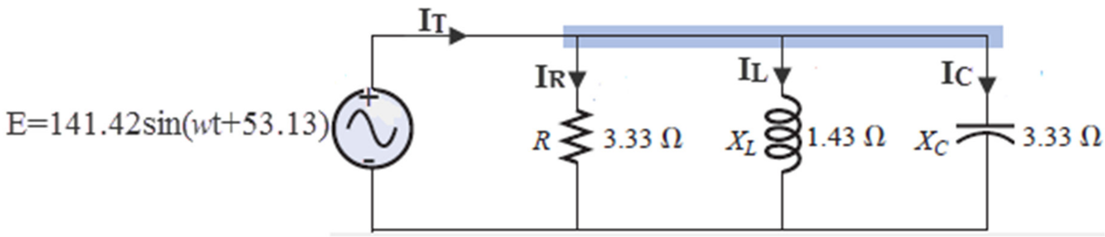
$Y_T = G + B_L + B_C = 0.2 - j0.125 + j0.05 = 0.2 - j\,0.075 = 0.2136\ \angle -20.56°$ S

$Z_T = 1/ Y_T = 4.68\ \angle 20.56°\ \Omega$

or

$Z_T = (R\ X_L\ X_C) / ((R\ X_L)+(R\ X_C)+(X_L\ X_C)) = 4.68\ \angle 20.56°\ \Omega$.(inductive circuit)

Example : For the network of Fig.below find Y_T , Z_T , I_R , I_L , I_C , and I_T .



Solution :

$E= (141.42/\sqrt{2}\ ) \angle 53.13^o = 100\angle 53.13^o$ V

$Y_T = G + j\ (B_C - B_L)$

$=(1/ 3.33)+(1/ j1.43)+(1/ -j3.33) = 0.3 – j0.7 + j3 = 0.3 – j0.4 = 0.5\angle -53.13^o$ S.

$Z_T = 1/ Y_T = 2\angle 53.13^o\ \Omega$.

$I_T = E\ Y_T = E / Z_T = (100\angle 53.13^o)(0.5\angle -53.13^o) = 50\angle 0^o$ A.

$I_R = E\ G = 30\ \angle 53.13°$ A

$I_L = E\ B_L = 70\ \angle -36.87°$A

$I_C = E\ B_C = 30\angle 143.13°$ A

$I_T = I_R + (I_C + I_L) = (18+j24)+((56- j42)+(-24+j18)) =(30\angle 53.13^o) +(40\angle -36.87°)$

$= 50\ \angle 0^o$ A.

$I_L > I_C$ (inductive circuit)

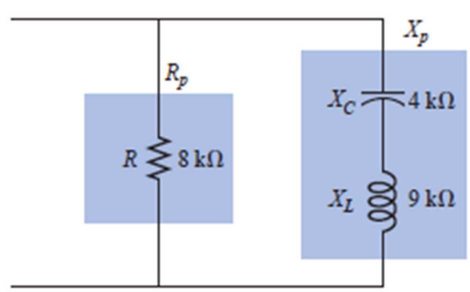
$P.F = \cos\varphi = \cos(\tan^{-1}\frac{I_L+I_C}{I_R}) = \cos (\tan^{-1}\frac{40}{30}) = \cos 53.13 = 0.6$ lagging.

$G = Y_T \cos\varphi$

$\cos\varphi = G / Y_T = 0.3/ 0.5 = 0.6$ lagging.

$\cos\varphi_z = \cos 53.13 = 0.6$ lagging.

Example : Determine the series equivalent circuit for the network of Fig.below.

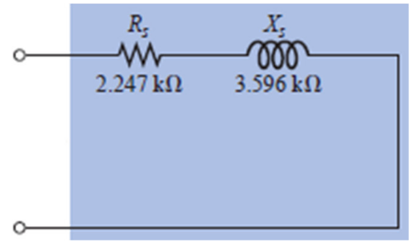


Solution :

$Z = R_P\ X_P / (R_P + X_P) = \frac{8K\times j5K}{8K+j5K}$

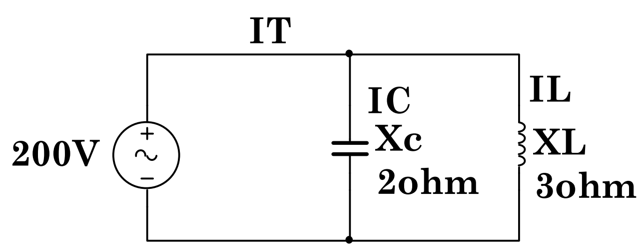
$= 2.247K+j3.596K\ \ \Omega$

inductive circuit



Note :
The methods were discussed in detail for dc circuits in Chapter 3, this chapter will consider the variations required to apply these methods to ac circuits.

Example : In the circuit shown in fig.find the total current flow in the circuit.

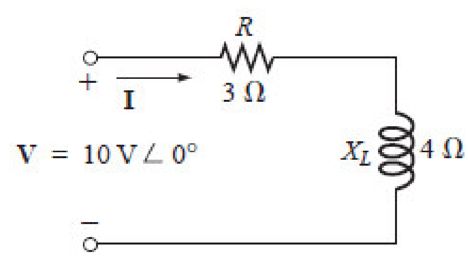


Solution :
$I_C = V\,B_C = V/ X_C = 200 / 2 = 100A.$ (I_C lead V)
$I_L = V\,B_L = V / X_L = 200 / 3 = 66.667A.$ (V lead I_L)

$I_T = I_C - I_L = 100 - 66.667 = 33.333A.$ (I_T lead V)

By using polar form
$X_C = -j\,2 = 2\angle -90^o\,\Omega$
$X_L = j\,3 \;\; = 3\angle 90^o\,\Omega$
$V = 200\angle 0$ V
$I_C = 200 / 2\angle -90^o = 100\angle 90^o$ A
$I_L = 200 / 3\angle 90^o = 66.667\angle -90^o$ A
$I_T = I_C + I_L = 100\angle 90^o + 66.667\angle -90^o = 33.333\angle 90^o$ A

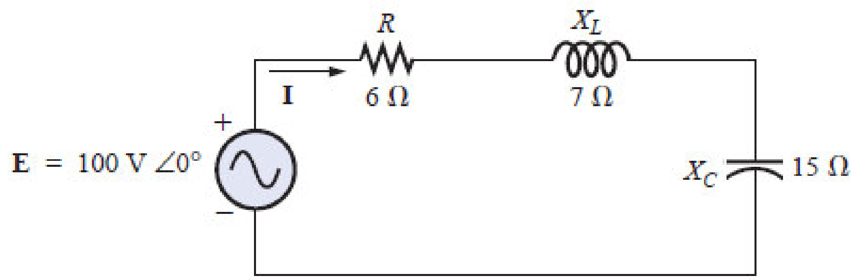
Example : The simple *R-L* circuit of Fig.find S.



Solution :
$S = V\,I^*$
$I = 10/(3+j4) = 2\angle -53.13^o$ A
$S = 10 \times 2\angle 53.13^o$
$\;\;= 20\angle 53.13^o = 12 + j16$ VA.
$P = I^2 R = 12$ W
$Q_L = I^2 X_L = 16$ VAR

P.F = Cos φ = P / S = 12/ 20 = 0.6 lagging (inductive circuit)

Example : Find the total number of watts, volt-amperes reactive, and voltamperes,and the power factor *Fp* for the network of Fig.shown below.



Solution :

$Z = 6 + j(7 - 15) = 6 - j8 = 10\angle -53.13^o\ \Omega$ (capacitive circuit)

$I = E / Z = 10\angle 53.13^o$ A

$S = E\,I^* = 100 \times 10\angle -53.13 = 1000\angle -53.13$ VA

$P = I^2 R = EI \cos\varphi = 600$ W

$Q = I^2 X = EI \sin\varphi = 800$ VAR

$S = P - jQ = 600 - j800 = 1000\angle 53.13$ VA

P.F = $\cos\varphi = P / S = 600 / 1000 = 0.6$ leading (capacitive circuit).

$V_R = I\,R = 6 \times 10\angle 53.13^o = 60\angle 53.13^o$ V

$V_L = IX_L = 7\angle 90 \times 10\angle 53.13^o = 70\angle 143.13$ V

$V_C = IX_C = 15\angle -90 \times 10\angle 53.13^o = 150\angle -36.87$V

Example:The voltage of a circuit $v=150\sin(\omega t + 10)$V and the current $i = 5\sin(\omega t - 50)$ A, Sketch the power triangle.

Solution :

$V = V_m / \sqrt{2} = 150\angle 10 / \sqrt{2} = 106\ \angle 10$ V

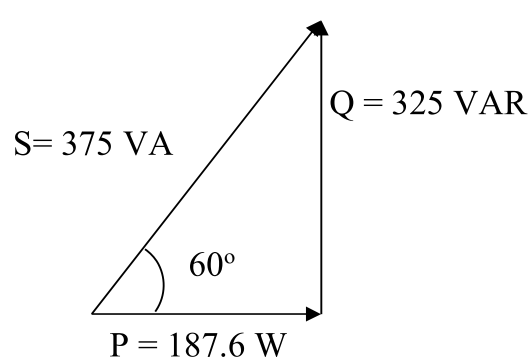
$I = I_m / \sqrt{2} = 5\angle -50 / \sqrt{2} = 3.54\angle -50$ A

$S = VI^* = (106\ \angle 10)(3.54\angle 50) = 375\angle 60 = 187.5 + j325$

$P = 187.6$ W

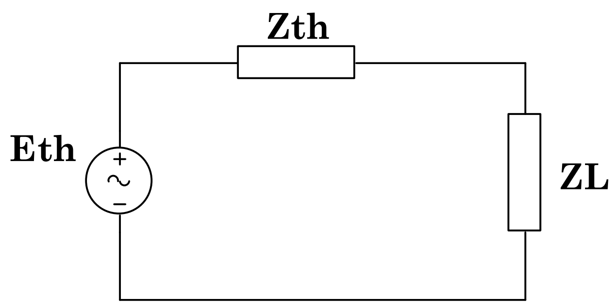
$Q = 325$ VAR

P.F= P/S = $\cos\varphi = 187.6 / 375 = 0.5$ lagging (inductive circuit)



5-4 Maximum Powe Transfer

maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.



$Z_L = Z_{Th}$ and $\varphi_{ZL} = -\varphi_{ZTh}$
$Z_{Th} = R \pm jX_{Th}$
$Z_L = R \mp jX_L$
$R_L = R_{Th}$ and $\mp j X_L = \pm j X_{Th}$

The conditions just mentioned will make the total impedance of the circuit appear purely resistive.

$Z_T = Z_{Th} + Z_L$
$Z_T = (R \pm j X_{Th}) + (R \mp j X_L)$
$Z_T = 2R$

Since the circuit is purely resistive, the power factor of the circuit under maximum power conditions is 1; that is,

P.F = 1 (maximum power transfer)

The magnitude of the current **I** of Fig. is

$I = E_{Th} / Z_T = E_{Th} / 2R$

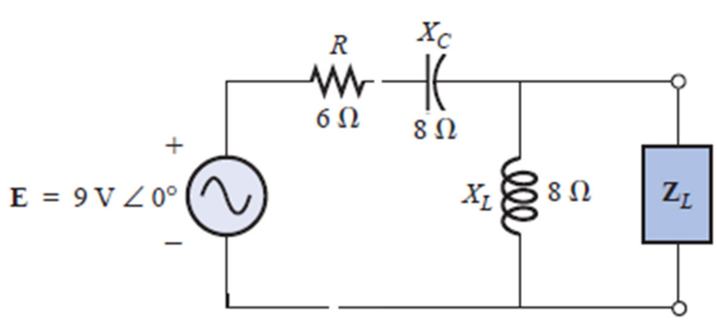
The maximum power to the load is

$P_{max} = I^2 R = (E_{Th} / 2R)^2 R$

and

$$P_{max} = \frac{E_{Th}^2}{4R}$$

Example : Find the load impedance in Fig. shown below for maximum power transfer to the load , and find the maximum power.



Solution :

$Z_{Th} = (j8)// (6 - j8) = ((j8)(6 - j8))/(6 + j8 - j8) = 80 \angle 36.87° / 6$

$= 13.33 \angle 36.87° = 10.66 + j\,8\,\Omega$

and $Z_L = 13.3 \angle -36.87° = 10.66 - j\,8\ \Omega$

To find the maximum power, we must first find $\mathbf{E}_{Th}$,as follows:

$E_{Th} = 9 (j8) /(6 + j8 - j8) = 72\angle 90 / 6 = 12\angle 90$ V

$P_{max} = \frac{E_{Th}^2}{4R} = 12^2 / 4(10.66) = 144/42.64 = 3.38$ W.

5-5 Power Measurment

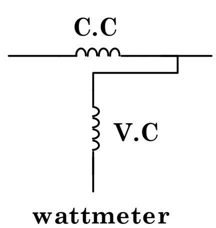
The power absorbed by a load is measured by an instrument called the *wattmeter*.

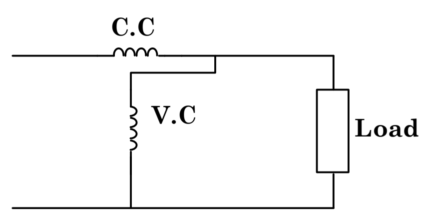
The wattmeter is the instrument for measuring the power in single phase or three phase system,and consists essentially of two coils ,the current coil and the voltage coil.A current coil with very low impedance (ideally zero) is connected in series with the load and responds to the load current. The voltage coil with very high impedance (ideally infinite) is connected in parallel with the load and responds to the load voltage.

- Measurement of power in single phase

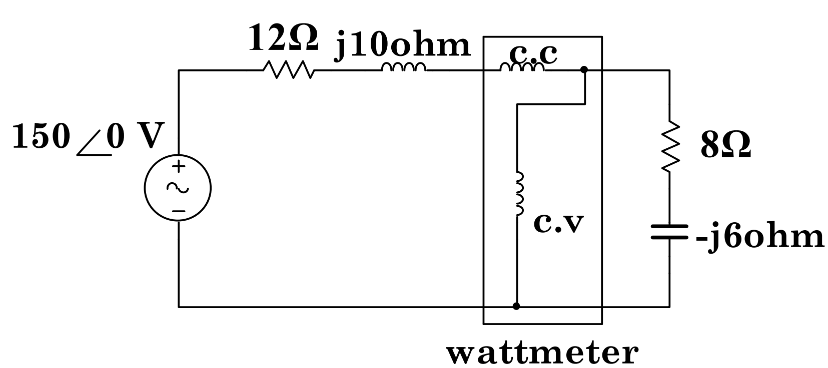
$P = VI \cos\varphi$

$= S \cos\varphi$





Example : Find the wattmeter reading of the circuit in Fig.shown below.



Solution :

$Z_T = 12 + j10 + 8 - j\,6 = 20 + j4 = 20.4\angle 11.3\ \Omega$ (inductive circuit)

$I = E/Z_T = 150\angle 0 / 20.4\angle 11.3 = 7.353\angle -11.3$ A

$V = 150(8 - j6) / (20+j4) = 73.54\angle -48.18$ V

$S = V I^*$

$= (73.54\angle -48.18)(7.353\angle +11.3) = 432.52 - j\,324.52$ VA.

The wattmeter reads

P = 432.52 W.

5-6 Power Factor Correction

Most industrial and domestic electrical loads operate at a required fixed a mount of real power P . The power factor thus becomes essentially important .

$P = VI\cos\varphi$

$I = P / V\cos\varphi$

From equation it is clear that for consatant P and V, the load current I can be reduced through increasing of φ .

Minimum current is therefore drawn from a supply when S = P and $Q_T = 0$.

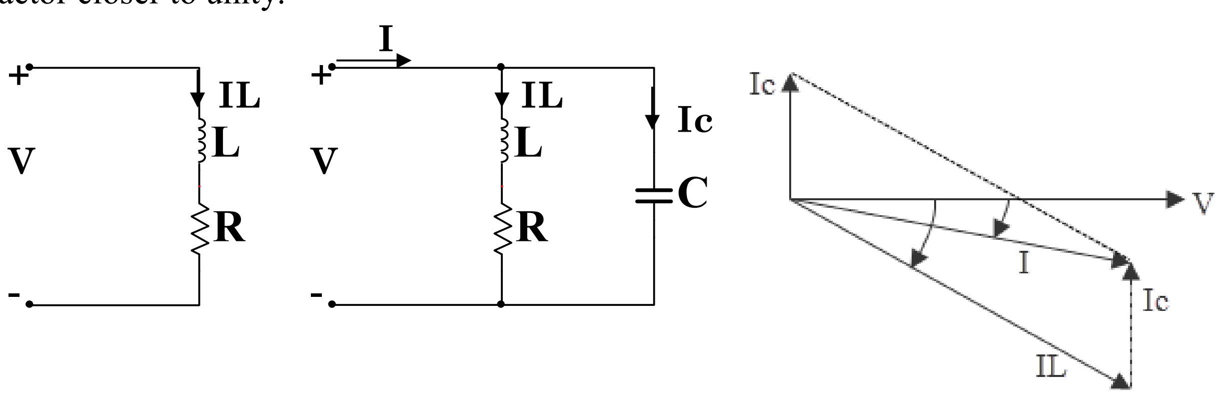
$0 < P.F < 1$

Q = 0, S = P and P.F = 1 When $\varphi = 0$

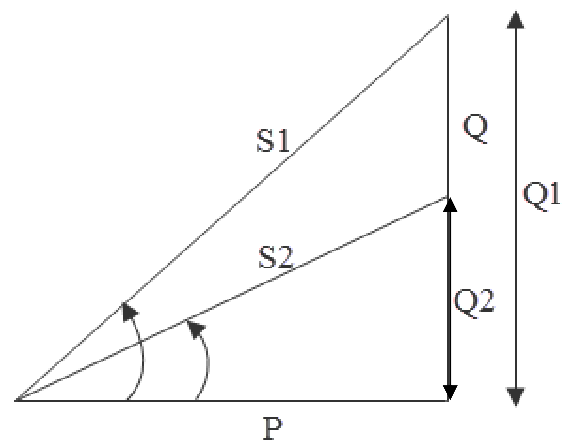
Note the power-factor angle approaches zero degrees and *pf* approaches1, revealing that the network is appearing more and more resistive at the input terminals.

When $\cos\varphi$ approaches 1 this means that the angle φ approaches zero.

Alternatively, power factor correction may be viewed as the addition of a reactive element (usually acapacitor) in parallel with the load in order to make the power factor closer to unity.



$P = S_1 \cos \varphi_1 = S_2 \cos \varphi_2$
$Q_1 = S_1 \sin \varphi_1 = P \tan \varphi_1$
$Q_2 = S_2 \sin \varphi_2 = P \tan \varphi_2$



The reduction in the reactive power is caused by the shunt capacitor, that is,
$Q = Q_1 - Q_2 = P(\tan \varphi_1 - \tan \varphi_2)$
$Q = V^2_{rms} / X_{C \text{ or } L}$

$Q_C = V^2_{rms}/X_C = \omega C V^2_{rms}$.
$C = Q_C / \omega V^2_{rms} = P(\tan \varphi_1 - \tan \varphi_2) / \omega V^2_{rms}$

$Q_L = V^2_{rms} / X_L = V^2_{rms} / \omega L$
$L = V^2_{rms} / \omega Q_L$

If Q is negative ($Q = -j\, Q_C$) where $Q = V^2 / X_C = V^2_{rms} WC$
If Q is positive ($Q = j\, Q_L$) where $Q = V^2/X_L = V^2_{rms} / WL$

Example : When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

Solution: $Q_C = Q_1 - Q_2$

first : Pf = 0.8 lag or $\cos\varphi_1 = 0.8 \Rightarrow \varphi_1 = 36.87°$
P=4kw =4000w

$S_1 = P/\cos \varphi_1 = 4000/0.8 = 5000$ VA=5 KVA

$Q_1 = S_1 \sin \varphi_1 = 5000 \sin 36.87 = 3000$ VAR = 3KVAR

second : Pf = 0.95 or $\cos \varphi_2 = 0.95 \Rightarrow \varphi_2 = 18.19°$

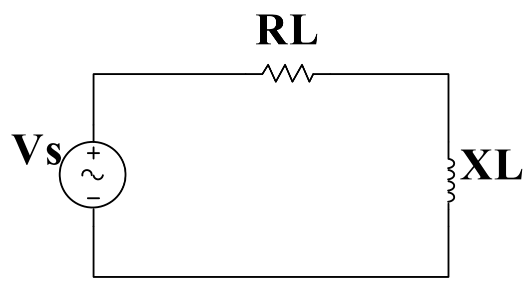
$S_2 = P/\cos \varphi_2 = 4000/0.95 = 4210.5$ VA=4.2105 KVA

$Q_2 = S_2 \sin \varphi_2 = 4210.5 \sin 18.19 = 1314.4$ VAR =1.3144 KVAR

$Q_C = Q_1 - Q_2 = 3000 - 1314.4 = 1685.6$ VAR
$Q_c = V^2_{rms} / X_C = V^2_{rms} WC = V^2_{rms}\, 2\pi f C \Longrightarrow C = Q_c / V^2_{rms}\, 2\pi f$

$C = \frac{1685.6}{2\pi \times 60 \times 120^2} = 310.5\mu F.$

Example : Calculate the complex power (the apparent power) for the circuit of Figure shown below and correct the power factor to unity by connecting a parallel reactance to the load Where V_S = 117□0 V ; $R_L = 50\ \Omega$; $jXL = j86.7\ \Omega$. Find
1. S = P + jQ for the complex load.
2. Value of parallel reactance required for power factor correction resulting in pf = 1.



Solution :

$Z = R + jXL = 50 + j86.7 = 100\angle 60^o\ \Omega$.

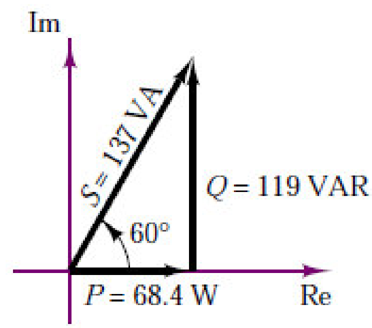
$I = V_S / Z = 117 / 100\angle 60^o = 1.17\angle -60^o$ A.

$S = V I^* = 117 \times 1.17\angle 60^o = (68.4 + j118.5)$ VA.

P = 68.4 W ; Q = 118.5 VAR

$\varphi = \tan^{-1}\frac{118.5}{68.4} = 60^o$

S= $137\angle 60^o$ VA.

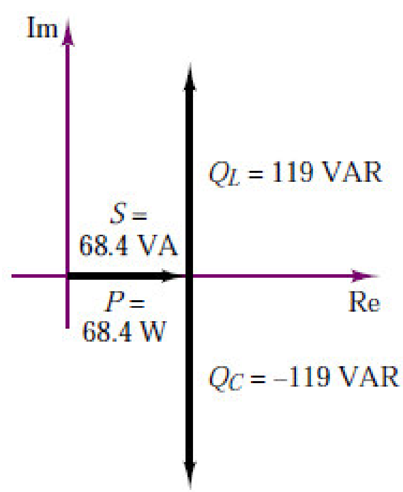


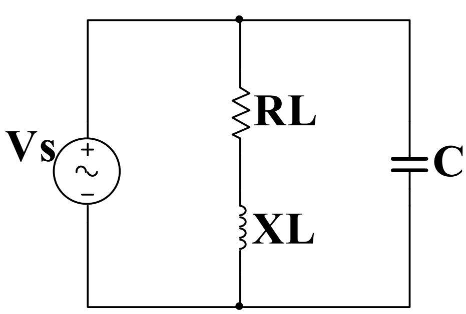
for the power factor correction, we observe that we need to contribute a negative reactive power equal to $-j118.5$ VAR.

$Q_C = -118.5$ VAR.

$XC = V^2 / Q_C = -(117)^2/118.5 = -j115\ \Omega$

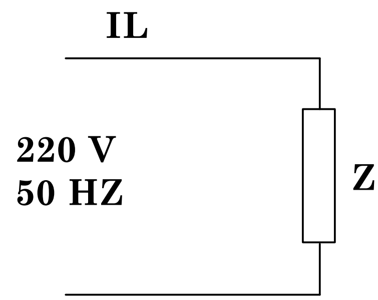
$C = 1/ W\ XC = 1 / (2\pi\times 60\times 115) = 23.066\ \mu F$.





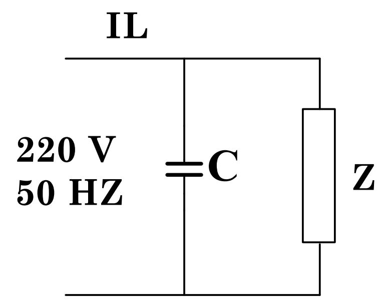
Parallel capacitor for unity power factor correction

Example : what will be the reduction the source current if a 50uf capacitor connected in parallel whith load drawn 6∠-45A .



Solution :

$I_L = 6\angle{-45}A$



$XC = -j / WC = -j\ 10^6 / 2\pi\times50\times50 = -j1000/5\pi$

$= -j\ 63.6\ \Omega$

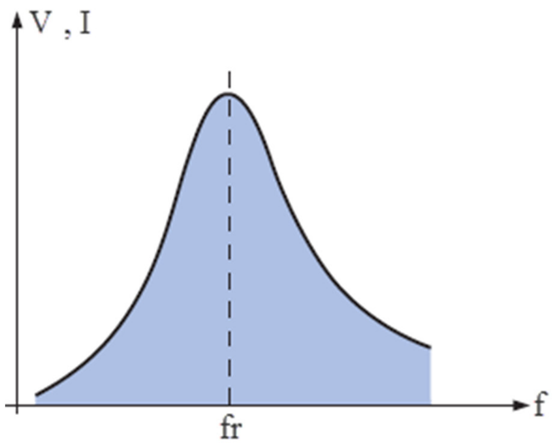
$Ic = V / XC = 220\angle0 / -j63.6 = j\ 3.45A = 3.45\angle90\ A$

$I_T = I_L + I_C = 6\angle{-45} + 3.45\angle90 = 4.31\angle{-10.5}\ A$

Reduction current = 6 – 4.31 =1.69 A , new pf = Cos10.5 =0.98 lag .

Chapter Six
Resonance

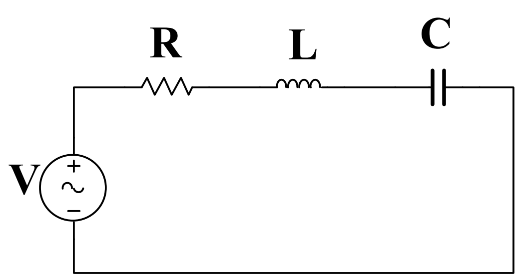
resonant circuits are able to pass a desired range of frequencies from a signal source to a load. In its most simple form, the **resonant circuit** consists of an inductor and a capacitor together with a voltage or current source.Although the circuit is simple, it is one of the most important circuits used in electronics. As an example, the resonant circuit, in one of its many forms,allows us to select a desired radio or television signal from the vast number of signals that are around us at any time.
The resonant circuit is a combination of *R,L,* and *C* elements having a frequency response characteristic similar to the one appearing in Fig. below .



Response curve of a resonant circuit.

Note in the figure that the response is a maximum for the frequency *fr*, decreasing to the right and left of this frequency.
In this chapter, we examine in detail the two main types of resonant circuits: the **series resonant circuit** and the **parallel resonant circuit.**

1- **series resonant circuit**



Because the circuit of Fig. below is a series circuit, we calculate the total impedance as follows:

$\mathbf{Z}_T = R + jX_L - jX_C$

$\quad = R + j(X_L - X_C)$

At resonant frequency

$$X_L - X_C = 0 \implies X_L = X_C$$

And

$\mathbf{Z}_T = R$

The value of ω that satisfies this condition is called the resonant frequency ωr. Thus, the resonance condition is $X_L = X_C$

$\omega_r L = 1 / \omega_r C$ and $\omega_r^2 = 1 / LC$

$$\omega_r = 1 / \sqrt{LC} \quad \text{or} \quad f_r = 1 / 2\pi \sqrt{LC}$$

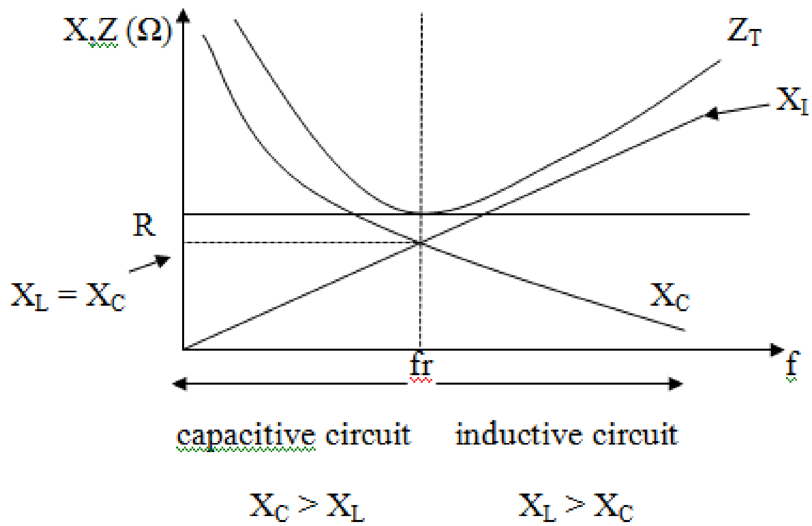
The total apparent power is equal to the active power dissipated by the resistor since $Q_L = -Q_C$.

The power factor of the circuit at resonance is Cos $cos\varphi = P / S = 1$

And $\varphi z = 0$

And $V_L = -V_C$

In this curve Placing the frequency response of the inductive and capacitive reactance of a series R-L-C circuit on the same set of axes.And Z_T versus frequency for the series resonant circuit.



The Quality Factor (*Q*):

The **quality factor** *Q* of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance; that is,

Q_r = reactive power / average power

$= I^2 X_L / I^2 R$ or $I^2 X_C / I^2 R$

$= X_L / R$ or X_C / R

$= \omega_r L / R$ or $1 / \omega_r C R$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

$|V_C| = |V_L| = Qr\ V$ at resonance

The Resonance Curve (Frequency Response):

We define the **bandwidth, BW**, of the resonant circuit to be the difference between the frequencies at which the circuit delivers half of the maximum power. The frequencies f1 and f2 are called the **half-power frequencies**, the **cutoff frequencies**,or the **band frequencies**.

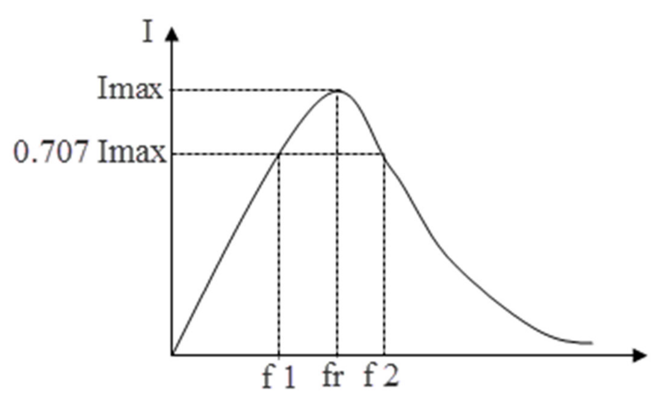
$BW = f_2 - f_1 = R / 2\pi L = f_r / Qr$ HZ

$f_r = \sqrt{f_1 f_2}$

for high-Q circuits ($Q \geq 10$)

$f_1 \cong f_r - (BW/2) \cong f_r - (R / 4\pi L)$

$f_2 \cong f_r + (BW/2) \cong f_r + (R / 4\pi L)$



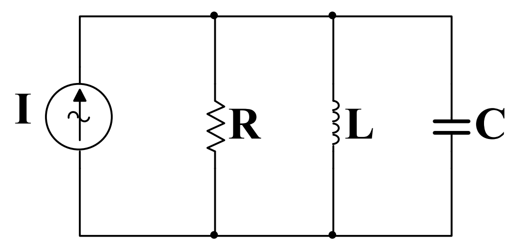
$\omega_r=2\pi f_r$, $\omega_1=2\pi f_1$ and $\omega_2=2\pi f_2$
f_1 : is lower frequancy
f_2 : is upper frequancy

2- parallel resonant circuit.

The parallel *RLC* circuit in Fig. shown is the dual of the series *RLC* circuit. So we will avoid needless repetition. The admittance is

$$Y = G + j\,B = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right) = \frac{1}{R} + j\left(wc - \frac{1}{WL}\right)$$

Resonance occurs when the imaginary part of **Y** is zero,



$$wc - \frac{1}{WL} = 0$$

or

$$\omega_r = \frac{1}{\sqrt{L\,C}} \text{ rad/s} \quad \text{or} \quad f_r = \frac{1}{2\pi\sqrt{L\,C}}$$

The Quality Factor (Q):

$$Q = \frac{\text{reactive power}}{\text{average power}} = \frac{V^2/X_L}{V^2/R} = \frac{R}{X_L} = \frac{R}{X_C} = = R\sqrt{\frac{C}{L}}$$

Bandwidth, BW :

$BW = f_2 - f_1 = 1 / RC = R / 2\pi L = f_r / Qr$ HZ

$f_r = \sqrt{f_1 f_2}$

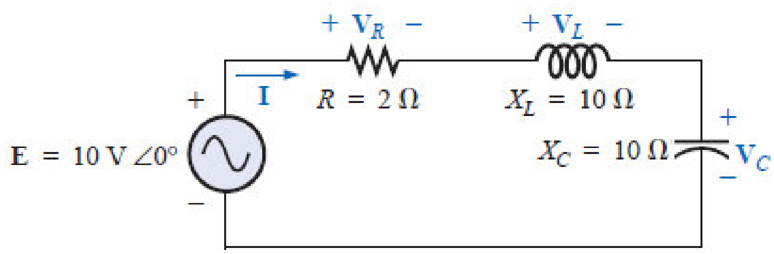
for high-Q circuits ($Q \geq 10$)

$f_1 \cong f_r - (BW/2) \cong f_r - (R / 4\pi L)$
$f_2 \cong f_r + (BW/2) \cong f_r + (R / 4\pi L)$

$\omega_r=2\pi f_r$, $\omega_1=2\pi f_1$ and $\omega_2=2\pi f_2$

$|I_C| = |I_L| = Q_l\, I$

Example :
a. For the series resonant circuit of Fig. below, find **I**, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ at resonance.
b. What is the Qr of the circuit?
c. If the resonant frequency is 5000 Hz, find the bandwidth.
d. What is the power dissipated in the circuit at the half-power frequencies?



Solution :

E = 10∠0 V

Z = 2 + j(10 – 10) = 2∠0 Ω

a- I = E / Z = 5∠0 A

V_R = **E** = 10 V ∠0° V

V_L = (I ∠0°)(X_L∠90°) = (5 ∠0°)(10 ∠90°) = **50 ∠90°** V

V_C = (I ∠0°)(X_C∠ −90°) = (5 ∠0°)(10 ∠ −90°) = **50 ∠ −90°** V

b- Qr = X_L / R = 10 / 2 = 5

c. BW = $f_2 – f_1$ = fr / Qr = 5000 /5 = **1000 Hz**

d- $P_{HPF} = (0.707\ I_{max})^2 R = 0.5\ I_{max}{}^2 R = P_{max} / 2 = I^2 R / 2 = [5^2 \times 2] / 2 =$ **25 W**

Example : A series *R-L-C* circuit has a series resonant frequency of 12,000 Hz.

a. If R = 5 Ω, and if X_L at resonance is 300 Ω, find the bandwidth.

b. Find the cutoff frequencies.

Solutions:

a- Qr = X_L / R = 300 / 5 = 60 (Qr >10)

BW = f_r / Qr = 12000 / 60 = 200 HZ

b- Since Qr ≥ 10, the bandwidth is bisected by fr. Therefore,

$f_1 \cong f_r$ - (BW/ 2) ≅ 12000 – 100 = 11900 HZ

$f_2 \cong f_r$ + (BW/ 2) ≅ 12000 + 100 = 12100 HZ

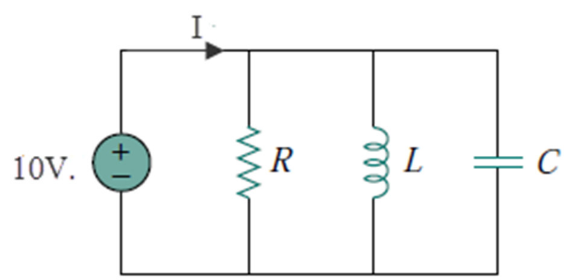
Example : In the parallel RLC circuit in Fig. below, let R = 8 KΩ. L = 0.2 mH, and C = 8 μF. (a) Calculate f_r, Q, and BW. (b) Find f_1 and f_2. (c) Determine the power dissipated at f_r, f_1, and f_2.

Solution :

a- $f_r = \dfrac{1}{2\pi\sqrt{0.2\times10^{-3}\times8\times10^{-6}}} = \dfrac{1}{2\pi\sqrt{1.6\times10^{-9}}}$ = 3978.87 HZ

Qr = R / X_L = 8000 / (2π×3978.87×0.2×10^{-3}) = 1600

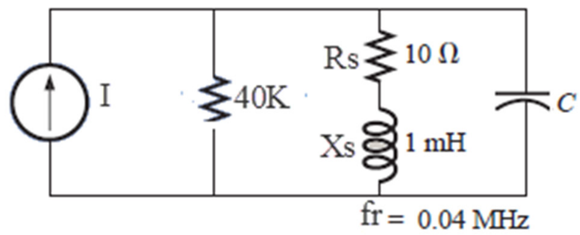
BW = f_r / Qr = 2.4868 HZ



b- Due to the high value of Q, we can regard this as a high-Q circuit.Hence,

$f_1 \cong f_r$ - (BW/ 2) $\cong$ 3978.87 – (2.4868/ 2) $\cong$ 3977.6266 HZ

$f_2 \cong f_r$ + (BW/ 2) $\cong$ 3978.87 + (2.4868/ 2) $\cong$ 3980.1134 HZ



C- Z= 8KΩ

I = V / Z = 10 / 8000 = 1.25 mA.

Since the entire current flows through *R* at resonance, the average power dissipated at f = fr is

$P_{HPF} = P_{max} / 2 = I^2 R / 2 = 6.25$ mW.

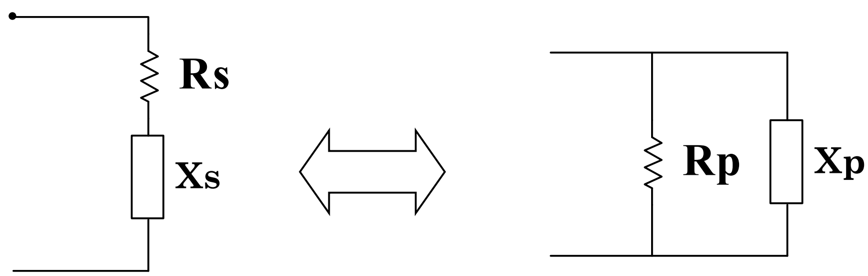
At f = f_1 , f_2

the current at this frequency is

$I = I / \sqrt{2} = 0.7071\ I$

$P = (0.7071\ I)^2 R / 2 = 3.215$mW .= $P_{HPF} / 2 = P_{max} / 4$

3-**Series-to-Parallel Conversion**



$Z_T = R_S + jX_S$

$$Y_T = 1 / Z_T = \frac{1}{R_S+jX_S} = \frac{(R_S-jX_S)}{(R_S+jX_S)\,(R_S-jX_S)} = \frac{R_S}{R_S^2+X_S^2} - j\frac{X_S}{R_S^2+X_S^2} = G - j\,B$$

$$= \frac{1}{R_P} - j\frac{1}{X_P}$$

$$R_P = \frac{R_S^2+X_S^2}{R_S} = R_S\,(1 + (X_S / R_S)^2) = R_S\,(1 + (Q_S)^2)$$

$$X_P = \frac{R_S^2+X_S^2}{X_S} = X_S\,(1 + (R_S / X_S)^2) = X_S\,(1 + (\frac{1}{Q_S^2}))$$

$Q_P = R / X_P$, $Q_S = X_S / R_S$

if the Q of the network is large (Q ≥ 10).

$R_P = Q_S^2\, R_S$

$X_P = X_S$

Example : For the network of Fig. below with fr provided:

a. Determine Q_S.

b. Determine R_p and X_P.
c. Calculate Z_{Tp}.
d. Find C at resonance.
e. Find Q_P.
f. Calculate the BW and cutoff frequencies.

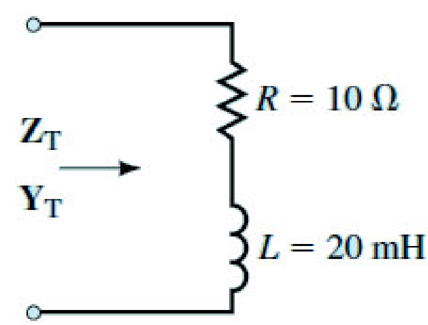
Solution :
a- $Q_S = X_S / R_S = (2\pi f_r L_S) / R_S = (2\pi\times40000\times10^{-3}) / 10 = 25.132$
b- $Q_S \geq 10$. Therefore,
$R_P = Q_S^2 R_S = (25.132)^2 \times10 = 6.31$ KΩ
$X_P = X_S = 2\pi\times40000\times10^{-3} = 251.32\ \Omega$
c- $Z_{Tp} = R\ //\ R_P = (40000\times6310) / 46310 = 5.45$ KΩ
d- $f_r = \frac{1}{2\pi\sqrt{L\,C}}$, $C = \frac{1}{(2\pi f_r)^2 L} = \frac{1}{(2\pi\times40000)^2\times10^{-3}} = 15.83\mu F$

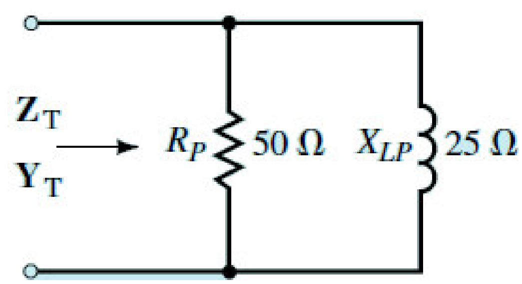
e - $Q_P = Q_r = R / X = 5.45$ KΩ / 251.32 Ω = 21.68
f – $BW = f_r / Q_r = 40000 / 21.68 = 1.85$ KHZ

$Q_r \geq 10$. Therefore,
$f_1 \cong f_r - (BW/2) \cong 40000 - (1850/2) \cong 39.075$ KHZ
$f_2 \cong f_r + (BW/2) \cong 40000 + (1850/2) \cong 40.925$ KHZ.

Example : For the series network of Figure shown below, find the Q of the coil at ω=1000 rad/s and convert the series RL network into its equivalent parallel network.

Solution :
$R_S = 10\Omega$, $X_{Ls} = \omega L = 1000\times20\times10^{-3} = 20\ \Omega$
$Q_S = X_{Ls} / R_S = 20 / 10 = 2$
$Q_S < 10$. Therefore,
$R_P = R_S(1 + Q_S^2) = 50\ \Omega$
$X_{LP} = X_S\left(1 + \left(\frac{1}{Q_S^2}\right)\right) = 25\ \Omega$





Example : Find the Q of the network of Figure shown below and determine the series equivalent.

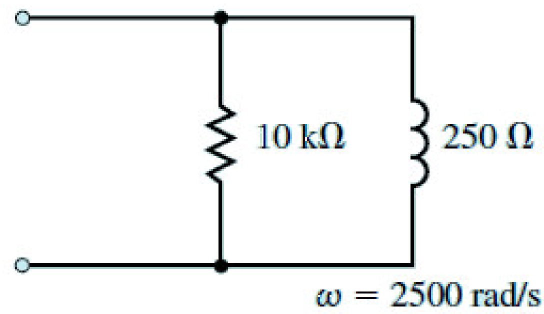
Solution :

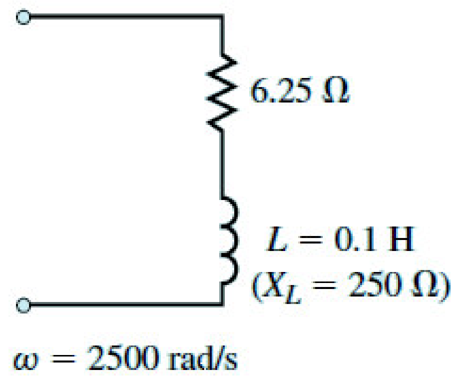
$Q_P = R_P / X_{LP} = 10000 / 250 = 40$

$Q_P \geq 10$. Therefore,

$R_S = R_P / Q_P^2 = 6.25\ \Omega$

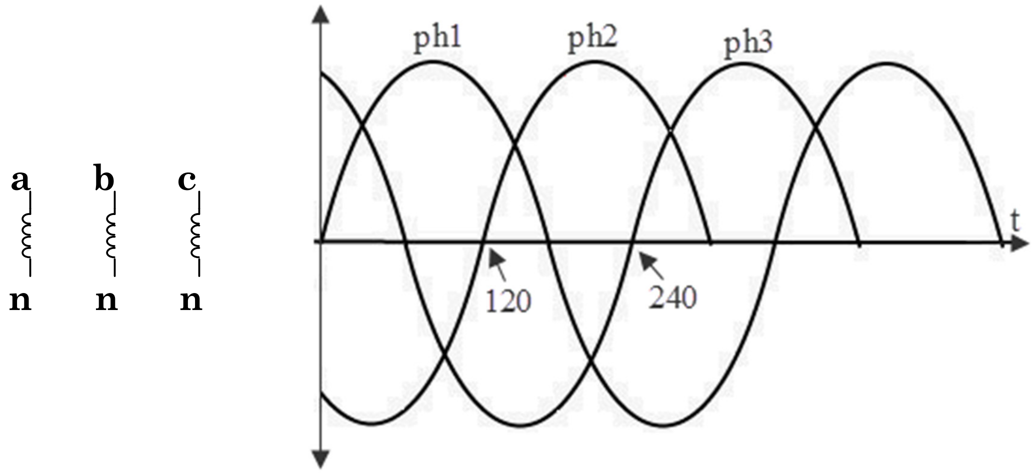
$X_{LS} = X_{LP} = 250\ \Omega$





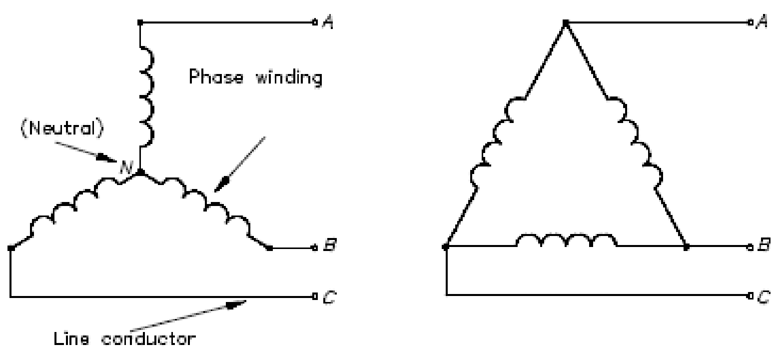
Chapter Seven
Three Phase Systems

A three-phase (3ϕ) system is a combination of three single-phase systems.



Advantage:
Three-phase equipment (motors, transformers, etc.) weighs less than single-phase equipment of the same power rating. They have a wide range of voltages and can be used for single-phase loads. Three-phase equipment is smaller in size, weighs less, which reduces the amount of copper required (typically about 25% less) and in turn reduces construction and maintenance costs. and is more efficient than single-phase equipment.

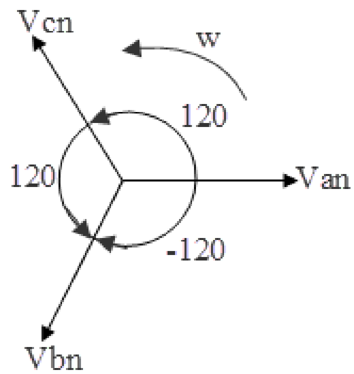
Three –Phase Connection:
Three-phase sources or load coccection as following:
(a) Y-connected ,
(b) Δ-connected .

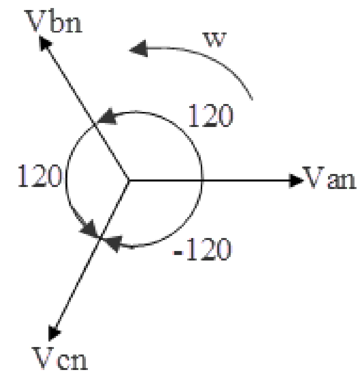


Since both the three-phase source and the three-phase load can be either wye- or delta-connected, we have four possible connections:

- Y-Y connection (i.e., Y- connected source with a Y- connected load).
- Y- Δ connection.
- Δ - Δ connection.
- Δ - Y connection.

Phase sequences:
(a) *abc* or positive sequence , (b) *acb* or negative sequence.





In a 3ϕ balanced system, power comes from a 3ϕ AC generator that produces three separate and equal voltages, each of which is 120° out of phase with the other voltages.

$V_{an} = V_p\angle 0^\circ$
$V_{bn} = V_p \angle -120^\circ$
$V_{cn} = V_p\angle -240^\circ = V_p\angle +120^\circ$

$V_{an} = V_p\angle 0^\circ$
$V_{cn} = V_p \angle -120^\circ$
$V_{bn} = V_p\angle -240^\circ = V_p\angle +120^\circ$

Note : These voltages are called phase voltages.

Exist in three phase system three voltages other is Vab,Vbc ,and Vca and called line voltage(line voltage the voltage between any two lines)

1-Y Connected:

$V_{ab} = V_{an} - V_{bn}$.
$V_{bc} = V_{bn} - V_{cn}$.
$V_{ca} = V_{cn} - V_{an}$.

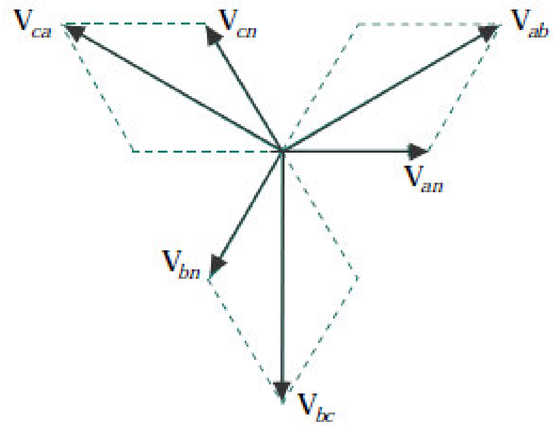
$$V_{ab} = V_{an} - V_{bn} = V_{ph} \angle 0 - V_{ph}\angle -120$$
$$= V_{ph} + j0 - V_{ph}\left[\cos(-120) + j\sin(-12)\right]$$
$$= V_{ph} - \left[-\tfrac{1}{2}V_{ph} - j\tfrac{\sqrt{3}}{2}V_{ph}\right]$$
$$= \tfrac{3}{2}V_{ph} + j\tfrac{\sqrt{3}}{2}V_{ph}$$
$$= V_{ph}\sqrt{(\tfrac{3}{2})^2 + (\tfrac{\sqrt{3}}{2})^2}\ \angle(\tan^{-1}\tfrac{\sqrt{3}}{1.5})$$
$$= \sqrt{3}\,V_{ph} \angle 30^0$$

$V_{bc} = \sqrt{3}\ V_{ph} \angle -90^0$
$V_{ca} = \sqrt{3}\ V_{ph}\angle -210^0$, $|V_L| = |\sqrt{3}\ V_{ph}|$

$I_{ph}=V_{ph}/Z$
$I_a = I_{ph}\angle 0^\circ$
$I_b = I_a \angle -120^\circ$
$I_c = I_a\angle -240^\circ = V_p\angle +120^\circ$
$I_{Nn}=I_{ph1}+I_{ph2}+I_{ph2}$

Same as line currents

$I_L = I_{ph}$



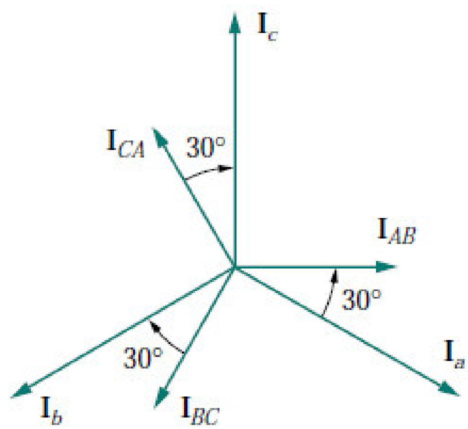
2-Δ Connected:

$I_a = I_{ab} - I_{ca} = \sqrt{3}\, Iph \angle -30^0$

$I_c = I_{ca} - I_{bc} = \sqrt{3}\, Iph \angle +120^0$

$I_b = I_{bc} - I_{ab} = \sqrt{3}\, Iph \angle -120^0$

$|I_L|=|\sqrt{3}\ I_{ph}|$



$V_L = V_{ph}$

$V_{ab} = V_{ph}\angle 0^\circ$
$V_{bc} = V_{ph}\angle -120^\circ$
$V_{ca} = V_{ph}\angle +120^\circ$

Same as phase voltages

Balanced system:

1-in star connection: Balanced phase voltages are equal in magnitude and are out of phas with each other by 120°,and

$I_{Nn}=0$,

$V_{an} + V_{bn} + V_{cn} = V_p\angle 0^\circ + V_p\angle -120^\circ + V_p\angle +120^\circ$
$= V_p(1.0 - 0.5 - j0.866 - 0.5 + j0.866) = 0$

$|V_{an}| = |V_{bn}| = |V_{cn}|$,

$Z_1 = Z_2 = Z_3 = Z_Y$

2-in delta connection: Balanced line currents are equal in magnitude and are out of phase with each other by 120°.

$I_L = |I_a| = |I_b| = |I_c|$, $I_{ph} = |I_{ab}| = |I_{bc}| = |I_{ca}|$

$Z_1 = Z_2 = Z_3 = Z_\Delta$