(Introduction)

Mechanics:- may be defined as that science which describes and predict the condition of rest or motion of bodies under the action of forces. It is divided into three parts:-

1- mechanics of rigid bodies.

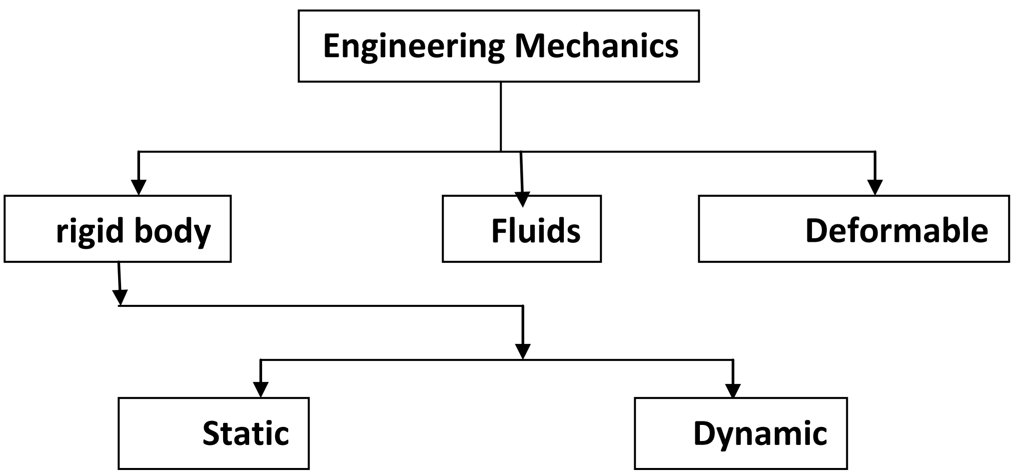
2-- mechanics of deformable bodies.

3- - mechanics of fluids.

mechanics of rigid bodies is sub divided in into:-

A- Statics

B- Dynamics



<u>Static</u>:- is the former dealing with bodies in rest.

<u>Dynamic</u>:- is the former dealing with bodies in motion.

In static of the study of mechanics, bodies are a assumed to be perfectly rigid(solid).

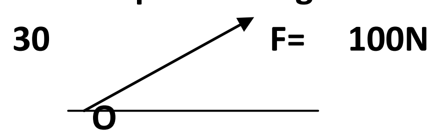
(Fundament principles in statics)

1- rigid body(solid):- is defined as the body which cannot effect under any external force.

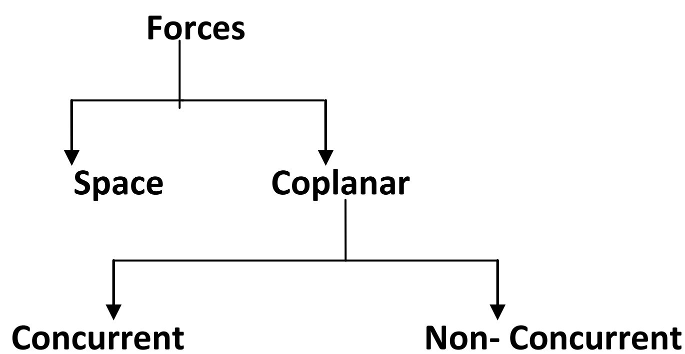
2- Force:- a force represents the action of one body on another which changes or tends to change the motion of the body acted on. Or Force The measure of the attempt to move a body. It is a fixed vector. For the rigid body problems or only the external effects of the magnitude, .external force onto the objects are of interested, force can be treated as a sliding direction, and line of action

3-Equlibrium:-A rigid body is said to be in equilibrium when the external forces acting on it equivalent to zero.

4-vector quantities:- Is defined as quantities which posses magnitude and direction (e.g. Force, Velocity, Moment)



5-Scalar quantities:- Is defined as quantities which posses only magnitude (e.g. Mass ,Density, Distance, length)



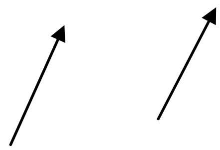
Unite of forces is in(S.I) unites is Newton(N) or Kilo Newton(KN)

The symbol of force is (F)

$F=m.g$ where the (m) in(kg) and(g) in (m/s^2)

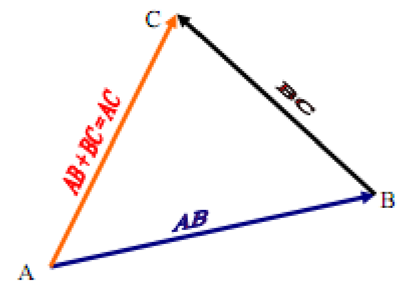
M= mass

Vector Addition

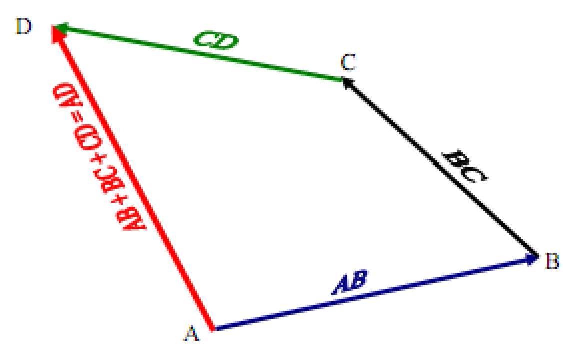


A B

AB+BC=AC



$$\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}+\overrightarrow{CD}=\overrightarrow{\mathrm{AD}}$$



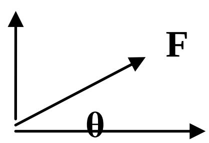
Resolution and compassion of a force:

Analysis of forces into a pair of perpendicular components is very important subject to be studied in order to have a full knowledge about the effect and distribution of forces on rigid bodies which are remaining of rest. For this reason I have designed this modular unit for this knowledge to be

understood.

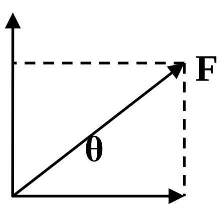
Let the force(F) shown in fig(1) with the direction(θ) we can resolve this force into two components:-

1- **Horizontal component (FX) which lies on X-axis**



2- **Vertical component (FY) which lies on Y-axis as shown in fig(2)**

Vertical component (FY)



Horizontal component (FX)

From fig(2):-

the horizontal component may be determine as:-

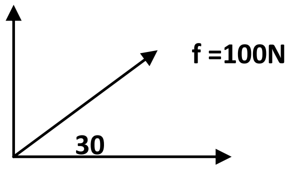
FX=F. cos θ

the vertical component may be determine as:-

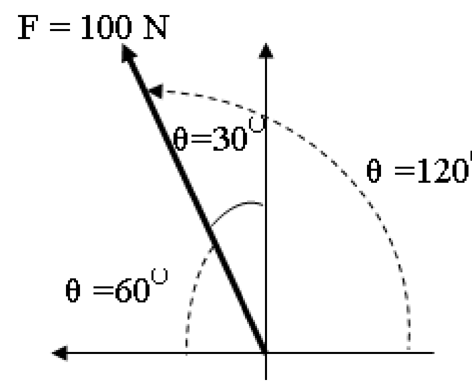
Fy=F sin θ

EX(1):- Find the two components of the force (100N) if θ=30,120,270

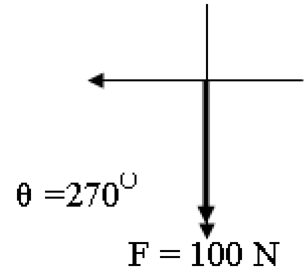
$\theta = 30^o$:-



Fx = F . cos θ
= 100 * cos 30
= 100 *
Fy = F . sin θ
= 100 * sin 30



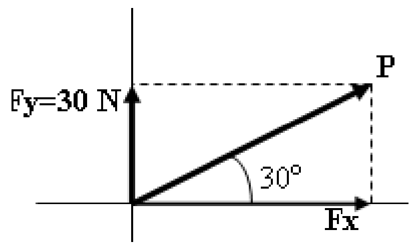
θ = 120o :
Fx = F . cos θ
= 100 * cos 120
= 100 * (- 0.5) = - 50 N
Fy = F . sin θ
= 100 * sin 120
= 100 *



θ = 270o :
Fx = F . cos θ
= 100 * cos 270
= 100 * (0) = 0
Fy = F . sin θ
= 100 * sin 270
= 100 * (- 1) = - 100 N

Example(2) :- The direction of the force (P) is (30O) , Find the horizontal component if the vertical component is (30 N) ?

Solution :



From the diagram shown :

Fy = 30 N

Fy = F . sin θ

30 = P * sin 30

30 = P * 0.5

P = 30 / 0.5 = 60 N

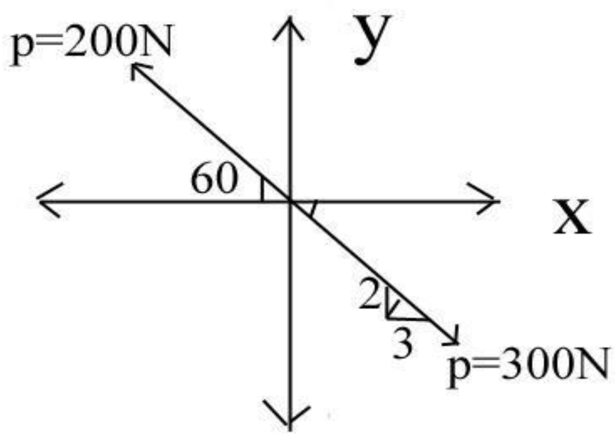
Fx = F . cos θ

= 60 * cos 30 = 60 *

2

3 = 30 3 N

Example: Determine the x and y components of the forces shown in the following figure:



Solution:

$P_X = P\cos\theta = 200\cos 60°$

$P_x = 200\,(0.5) = 100N$

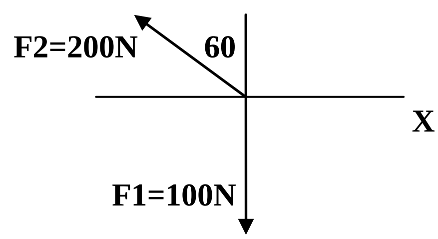
$P_y = p\sin\theta = 200\sin 60° = 200(0.866) = 172N$

$I = {}^{2}(2) + {}^{2}(3) = 4+9 = 13 = 3.61$

$F_x = f\cos\theta = 300\,(3/3.61) = 249\ N$

$F_y = f\sin\theta = 300\,(2/3.61) = 166\ N$

Example(3):- Find the horizontal and vertical components for the forces shown in figure.



Example(4):- the horizontal components for the force(F) shown in figure(60N) to the right through(O).determine the vertical component and magnitude of(F).

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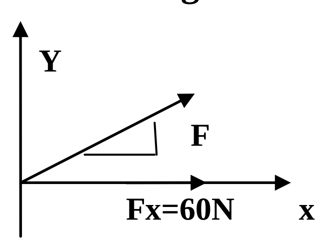


Resultant of force system

A simple force which can replace the original forces system without changing its external effect on a rigid body. the symbol of resultant force is(R)

The unit of resultant force is Newton(N)

In order to find resultant direction to solve problem. it must be follows the following procedure resolve all inclined forces to two horizontal and vertical components.

1-Find the resultant of horizontal forces by:-

RX=∑FX

2- Find the resultant of vertical forces by:-

Ry=∑Fy

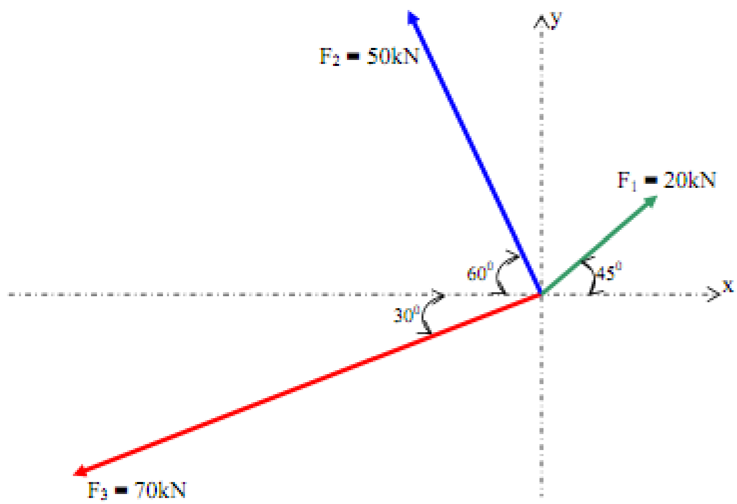
3- Find the resultant(R) by:-

$$R=\sqrt{RX^2+Ry^2}$$

4- Find the slope angle of the resultant by:-

$$\tan\theta=\frac{Ry}{Rx}$$

Example(1): Determine the resultant of the concurrent coplanar forces system as shown in the following figure.



Rx= 20 cos(45)-50 cos (60) -70 cos (30)

Ry= 20 sin(45)+50 sin (60) -70 sin (30)

Rx = F + F + F = 14.14 – 25 -60.62 = -71.48 KN

Ry = F + F + F = 14.14 + 43.30 -35 = 22.44 KN

R(Rx,Ry) = (-71.48 KN , 22.44 KN)

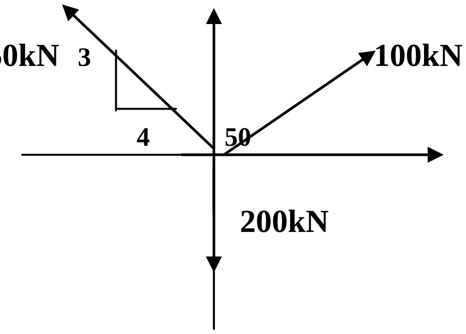
$R=\sqrt{Rx^2 + Ry^2}$

$R=\sqrt{-71.48^2+22.44^2}=74.92kN$

$\tan\theta = \frac{Ry}{Rx}$

$\theta = \tan^{-1}\frac{Ry}{Rx}(\frac{22.44}{71.48}) = 17.43°$

Example(2): Determine the resultant of forces system shown in figure.



solution:

$$R=\sqrt{Rx^2+Ry^2}$$

$Rx=\sum Fx$

$=100\sin 50-1504/5=43.4$

$Ry=\sum Fy$

$=100\cos 50+1503/5=45.73$

$$R=\sqrt{(43.4)^2+(45.73)^2}$$

$=63.04$

$$\theta=\tan^{-1}\frac{Ry}{Rx}\left(\frac{45.73}{43.04}\right)=46.49°$$

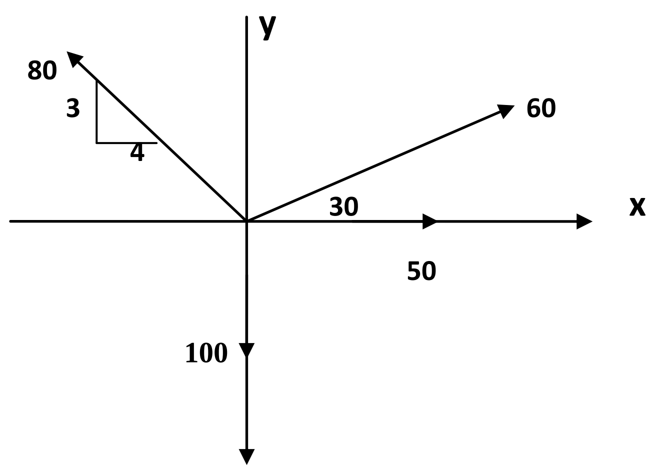
Example(3): Determine the resultant of forces system shown in figure.

Solution:-

Solution force(60):-

Fx=60cos30=51.9

Fy=60sin30=30



Solution force(80):-

$Fx=80\frac{4}{5}=64$

$Fy=80\frac{3}{5}=48$

$Rx=\sum Fx$

$51.9+50-64=37.9$

$Ry=\sum Fy$

$=-100+30+48=-22=22$

$$R=\sqrt{(37.9)^2+(22)^2}=43.82kN$$

$$\theta=\tan^{-1}\frac{Ry}{Rx}\left(\frac{22}{37{,}5}\right)=30.13°$$

Example(4): Determine the resultant of forces system shown in figure.

solution:

300kN

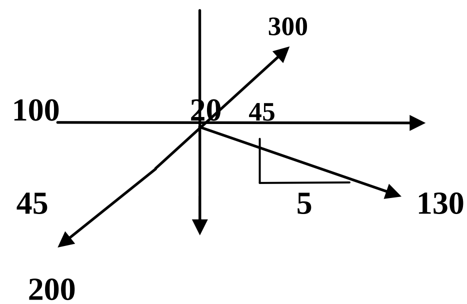
$$R=\sqrt{Rx^2+Ry^2}$$

$$Rx=\sum Fx$$

$$=300\cos45+130\frac{12}{13}-100-200\cos45=90.72$$

$$Ry=\sum Fy$$

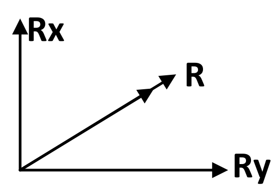
$$=300\sin45+20-130\frac{5}{13}-200\sin45=40.71$$



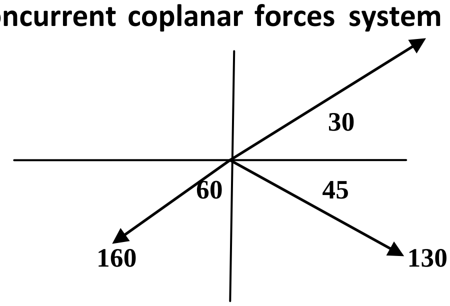
$$R=\sqrt{(90.72)^2+(40.71)^2}$$

$$=99.42$$

$$\theta=\tan^{-1}\frac{Ry}{Rx}\left(\frac{40.71}{90.72}\right)=24.17°$$



H.W :- Determine the resultant of the concurrent coplanar forces system as shown in the following figure.
200N



H.W :- Determine the resultant of the concurrent coplanar forces system as shown in the following figure

Moment of a force

The Moment of a force is a measure of its tendency to cause a body to rotate about a specific point or axis. This is different from the tendency for a body to move, or translate, in the direction of the force. In order for a moment to develop, the force must act upon the body in such a manner that the body would begin to twist. This occurs every time a force is applied so that it does not pass through the centroid of the body. A moment is due to a force not having an equal and opposite force directly along it's line of action.

The magnitude of the moment of a force acting about a point or axis is directly proportional to the distance of the force from the point or axis. It is defined as the product of the force (F) and the moment arm (d). The moment arm or lever arm is the perpendicular distance between the line of action of the force and the center of moments.

Moment = Force x Distance or M = (F)(d)

M = The Moment of a force

F = applied force

d = the perpendicular distance between the line of action of the force and the center of

The moment of a force about a point or axis measures of the tendency of the force to cause the body to rotate about the point or axis.

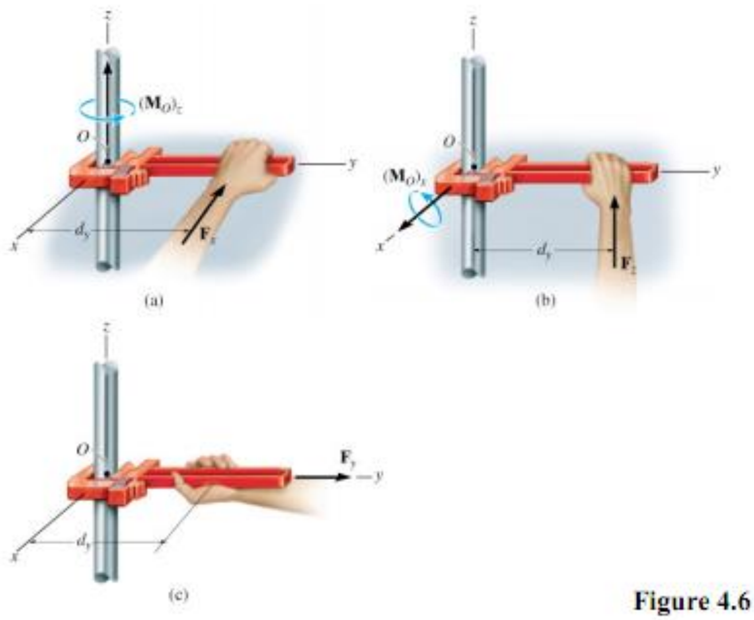


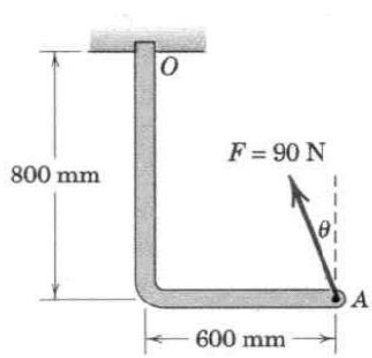
Figure 4.6

Magnitude of Moment

$$M_o = Fd$$

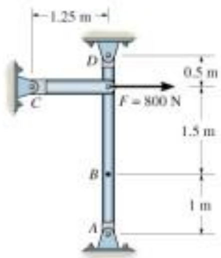
Where ***d*** is the moment arm or perpendicular distance from the axis at point *O* to the line of action of the force

Q:- Determine the moment of the(90N) force acting on the frame as shown in figure about points(A,O)



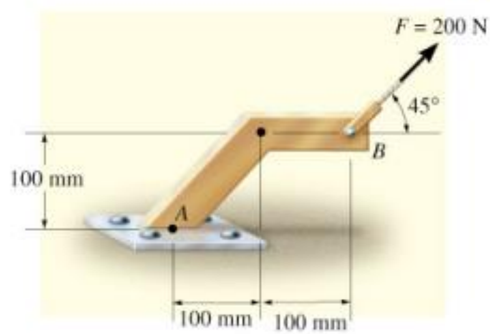
1. Determine the moment of the 800 N force acting on the frame in the figure below about points A, B, C, and D

(Ans: 2000 N·m, 1200 N·m, 0 N·m, 400 N·m)



1. A 200 N force acts on the bracket shown below. Determine the moment of the force about point A

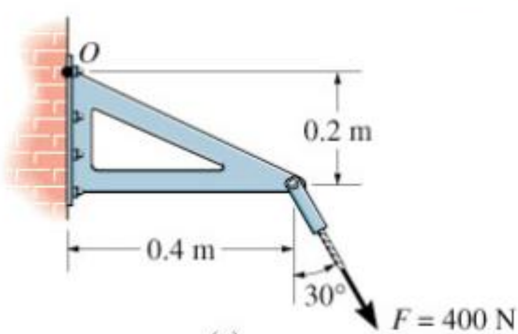
(Ans: 14.1 N·m)



(a)

2. The force F acts at the end of the angle bracket shown in the figure below. Determine the moment of the force about point O.

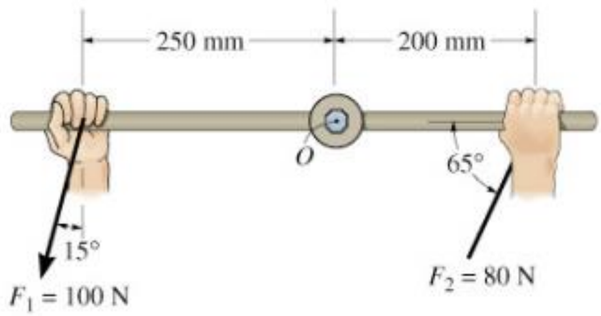
(Ans: 98.6 N·m)



(a)

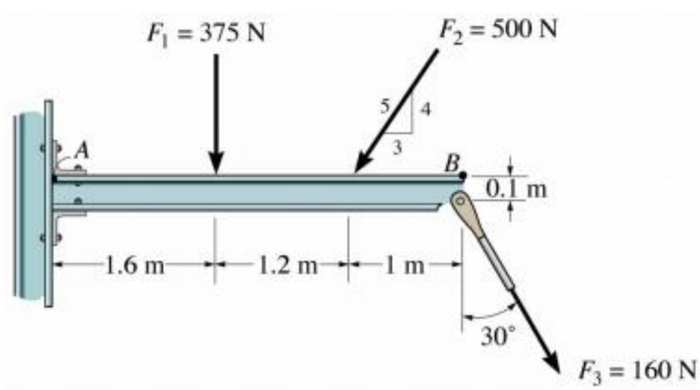
3. The wrench is used to loosen the bolt. Determine the moment of each force about the bolt's axis passing through point *O*.

(Ans: 24.1 N·m, 14.5 N·m)



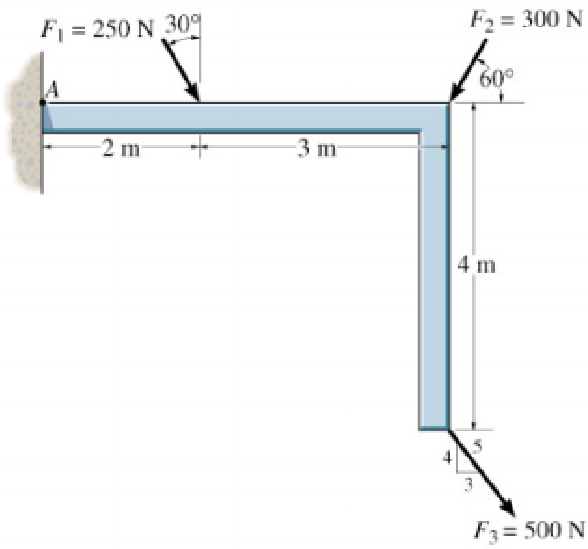
4. Determine the moment about point A of each of the three forces

(Ans: 600 N·m, 1.12 kN·m, 518 N·m)



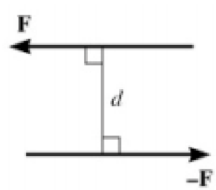
5. Determine the moment of each of the three forces about point A. Solve the problem first by using each force as a whole, and then by using the principle of moments.

(Ans: 433 N·m, 1.30 kN·m, 800 N·m)



Moment of a Couple

A couple is defined as two parallel forces that have the same magnitude, opposite directions, and are separated by a perpendicular distance d. Since the resultant force of the force composing the couple is zero, the only effect of a couple is to produce a rotation or tendency of rotation in a specified direction.



Magnitude of a Moment of a Couple

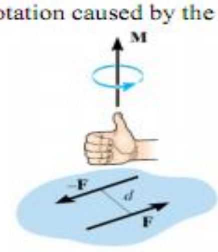
The magnitude of a couple moment is given as

$$M = Fd$$

Where d is the perpendicular distance or moment arm between the two parallel forces.

Direction of a Moment of a Couple

The direction and sense of a couple moment is determined using the right hand rule, where the thumb indicates the direction when the finers are curled with the sense of rotation caused by the two forces.

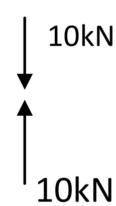


Equivalent Couples

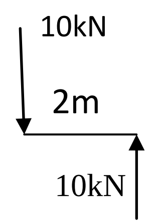
Two couples are said to be equivalent if they produce the same moment.

example:-Find the couple fore the figures:-

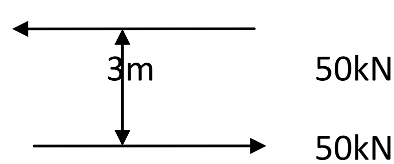
Mc=10*0=0



MC=10*2=20kN

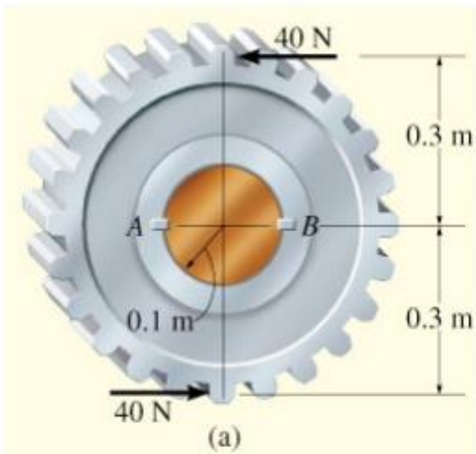


MC=50*3=150kN



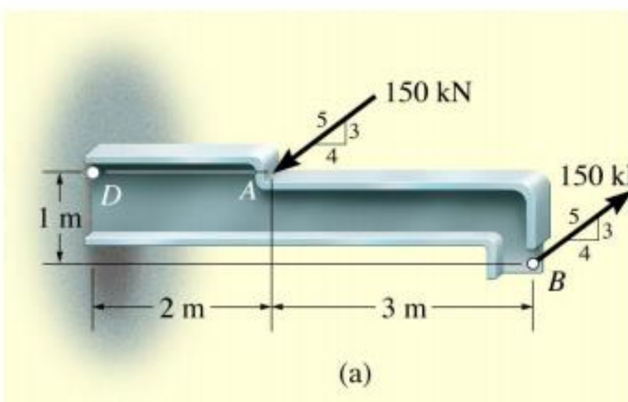
1. A couple acts on the gear teeth as shown in the figure below. Replace it by an equivalent couple having a pair of forces that act through (a) points A and B.

(Ans: 240)



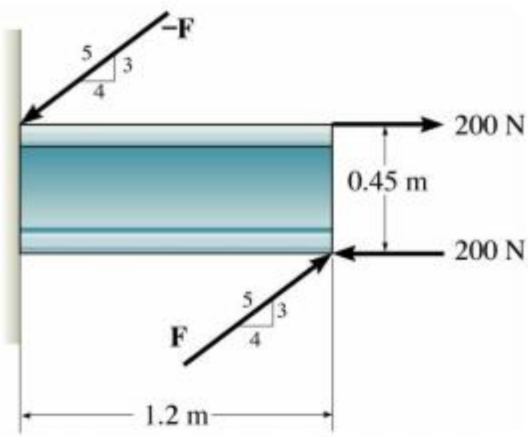
2. Determine the moment of the couple acting on the machine member shown below

(Ans: 390 N·m)



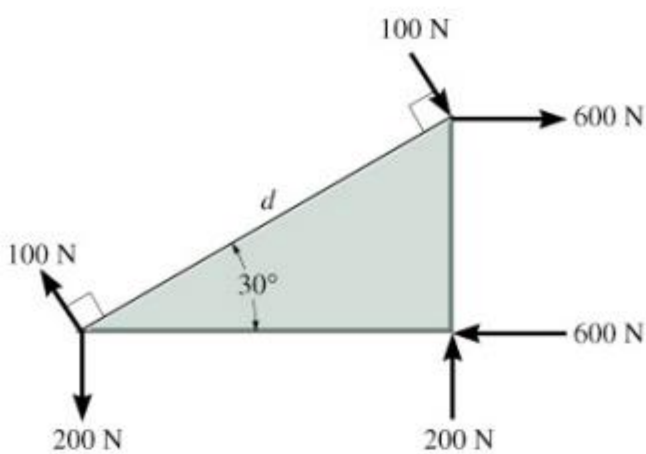
Two couple act on the beam as shown below. Determine the magnitude of F so that the resultant couple moment is 100 N·m counter clockwise. Where on the beam does the resultant couple act?

(Ans: 176 N·m)



The ends of the triangular plate are subjected to three couples. Determine the plate dimension d so that the resultant couple is 350 N·m clockwise.

(Ans: 1.54·m)



Equilibrium

In other words, for stationary objects or objects moving with constant velocity, the resultant force acting on the object is zero. The object is said to be in **equilibrium**.

If a resultant force acts on an object then that object can be brought into equilibrium by applying an additional force that exactly balances this resultant. Such a force is called the *equilibrant* and is equal in magnitude but opposite in direction to the original resultant force acting on the object.

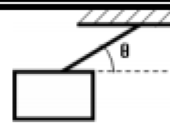
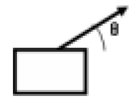
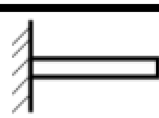
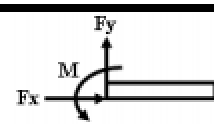
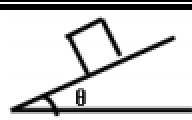
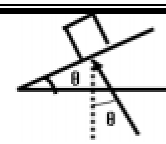
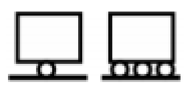
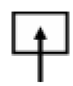
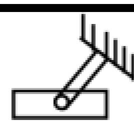
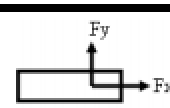
Equilibrium It's means that the body either at rest or move with constant velocity the condition of equilibrium in: .

(1) Concurrent Copland forces is R= 0 (Rx = o and Ry = 0)
(2) Non concurrent Copland forces is R = 0 and$\sum$ M = 0)

The following table shows how to represent the act of manytypes of contact upon the F.B.D:

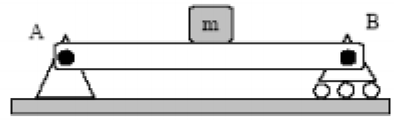
FREE BODY DIAGRAM

Free – body diagram and the mechanical effects

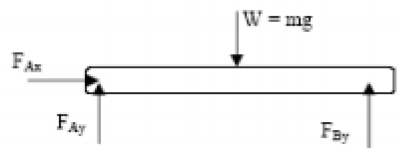
The name of the body	The effect of the body	Free-body diagram
Earth		
Flixible cables And ropes		
Cantilever beam		
Smooth surface		
Rollers , balls , cylinders		
Smooth pins		

Free body diagram : is a sketch to show all the forces and reactions acting on the body For example :

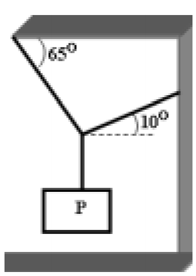
Mass at mid-point on beam (length L)



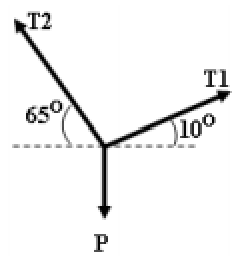
Free body diagram



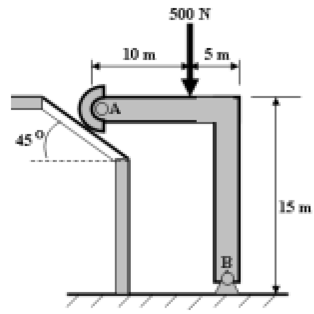
Draw Free – body diagram for
the ropes system shown in fig.



Solution



Ex (2) :
Determine the reactions at the points (**A**) and (**B**) , the angle beam was in equilibrium state as shown in fig .



Solution

Σ M (A) = 0

500* 10 – N * 15 = 0

N = 5000 / 15 = 333.34 N

Σ Fy = 0

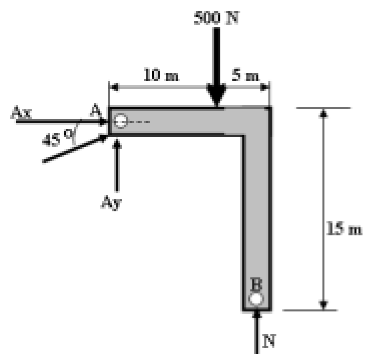
Ay + N – 500 = 0

Ay + 333.34 – 500 = 0

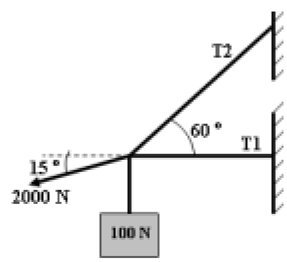
Ay = 166.67 N

Σ Fx = 0 , Ax = 0

RA = Ay = 166.67 N



Ex (3) :
Determine the tension forces (**T1**) and (**T2**) in the equilibrium system shown in fig. .



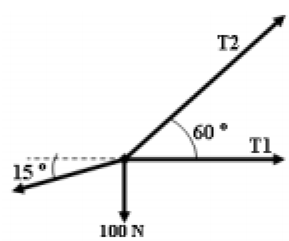
Solution

Σ Fx = 0

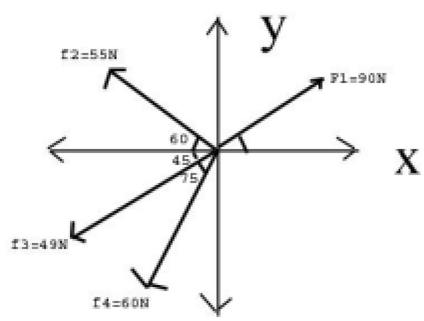
T1 . cos (0) + T2 .cos (60) – 2000 cos (15) = 0
T1 + 0.5 T2 – 1931.85 = 0 …….. (1)

Σ Fy = 0

T1 . sin (0) + T2 . sin (60) – 2000 . sin (15) – 100 = 0
0.866 T2 – 617.63 = 0
T2 = 713.2 N
Subs. in (1)
T1 = 1575 N



Ex(6):- Show analytically if the system of forces in the following figure is



<u>Solution:</u>

$Rx = \Sigma fx = F_1 \cos 30^{\circ} - f_2 \cos 60 - f_3 \cos 55 - f_4 \cos 75$

$Rx=90(0.866)-55(0.5)-49(0.707) - 60(0.258)$

$77.8-27.5-34.7-15.6-0$

$Ry = \Sigma fy = f_1 \sin 30^0 + f_2 \sin 60^0 - f_3 \sin 45^0 - f_4 \sin 75^0$

$Fy = 90\ (0.5) + 55(0.866) - 49(0.707) - 60(0.966)$

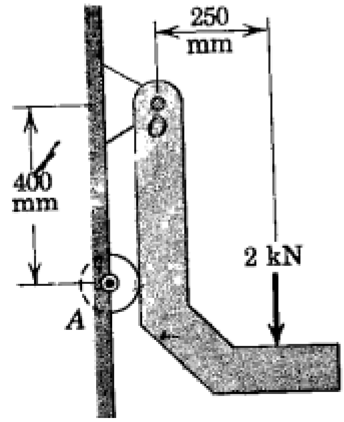
$Ry = o$

Since both $Rx = 0$ and $Ry = 0$ then the system of force is in equilibrium.

Ex (5) :

Find out the reaction on the cylinder (**A**)

and the total force acting on the pin (**O**)



Solution

Σ M (O) = 0

2 * 250 - R_A * 400 = 0

400 R_A = 500

R_A = 1.25 KN

Σ Fy = 0

Oy – 2 = 0

Oy = 2 KN

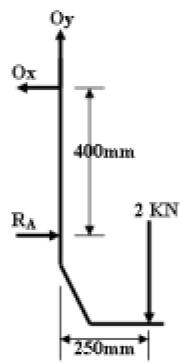
Σ Fx = 0

R_A – Ox = 0

Ox = R_A = 1.25 KN

$$F = \sqrt{(Ox)^2 + (Oy)^2}$$

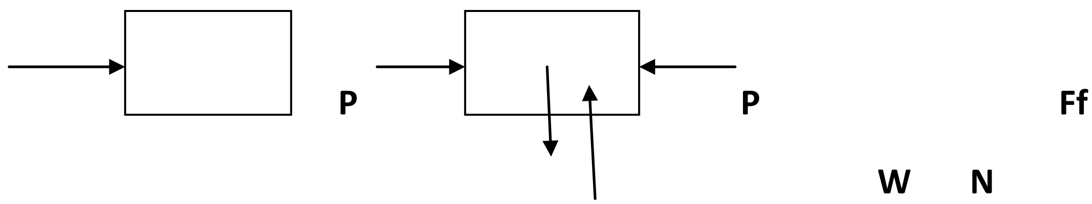
$$F = \sqrt{(1.25)^2 + (2)^2} = 2.35 N$$



F.B.D

Friction

F*riction* **may be defined as the contact resistance exerted by one body upon a second when the second body moves or tends to move past the first body.**



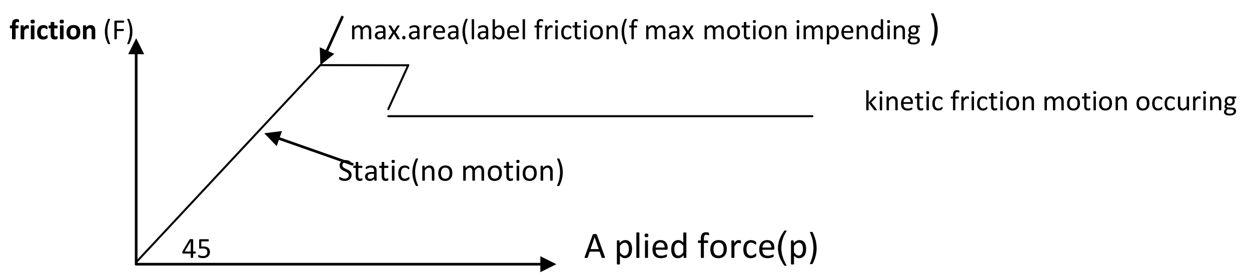
$$F = \mu \, . \, N$$

Where:

F = friction force

N =Normal reaction

μ = Coefficient of friction



μ_k: coefficient of kinetic friction

μ_s: coefficient of static friction

$\mu_s > \mu_k$

Dry vs. Wet Friction:

Friction occurs whenever two bodies are in contact with each other and are either moving with respect to each other or are due to forces acting on the bodies liable to move. When studying friction one has to distinguish between at least two major phenomena.

Dry Friction occurs when the surfaces in contact with each other are free of any lubricants. Motions of the two bodies in a direction parallel to the touching surfaces is prevented (or hampered) due to molecular adhesion and/or irregularities on the involved surfaces often too minute to discern with the necked eye.
In this course we are dealing only with dry friction.

Wet friction occurs when the surfaces of two solid bodies are not directly in contact but separated by a thin film of lubricants. Again friction will try to hamper the motion but the underlying physics is related to fluid mechanics which is not part of this course.

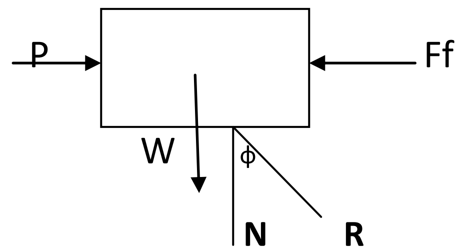
Above distinction is that of two extreme case with mixed friction in between which often occurs during start-up of motion when the separating lubricant film is not fully developed.

Angle of friction(Φ)

It's the angle between the total reaction (R) and its normal component, when limiting friction.

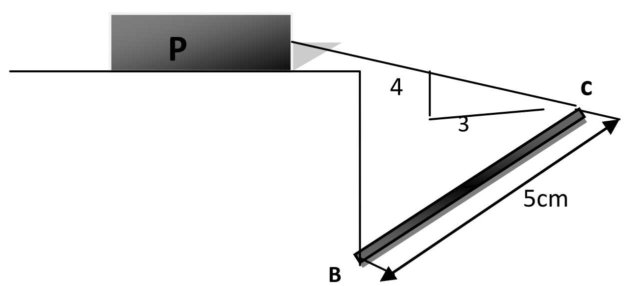
The tangent of this angle is equal to the coefficient of friction.

The angle of indication can be increased gradually, when the motion (impending) the angle of friction ϕ) equal the angle of incline (x) = x (impending motion).



Example:

In the following figure find the force (A)if coefficient of friction between the(150N) body(A) and the plane is (0.5).the bar (BC) weight(100N)



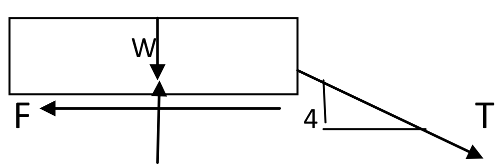
Solution:-

from bar (BC):-

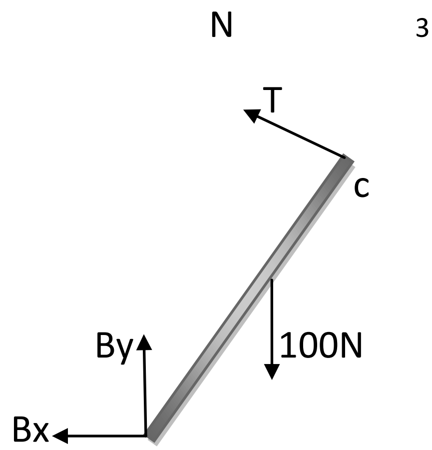
$\sum MB = 0$

$100(1.5) - T(1.5) = 0$

$T = 30$



from body(A):-



$\sum Fy = 0$

$N-50-30(\frac{3}{5})=0$

$N=68$

$\sum Fx = 0$

$30(\frac{4}{5})-F=0$

$F=24$

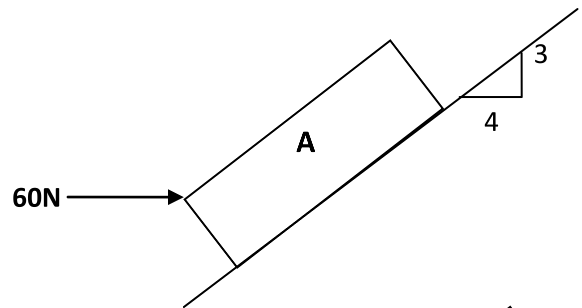
$F^{-}=\mu.N$

$=0.5*68=34$

$F < F^{-}$

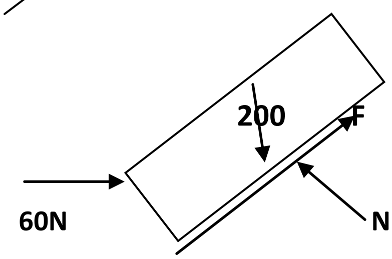
The body is equilibrium

Example:- Body (A) weight (200N). the coefficient of friction between body(A) and the inclined plane is (0.4).Determine the frictional force on the block.



Solution:-

Assume the body sliding down



∑Fy= 0

N-160-36=0

N=196 N

∑Fx= 0

F+48-120=0

F=72N

F^{-}=μ.N

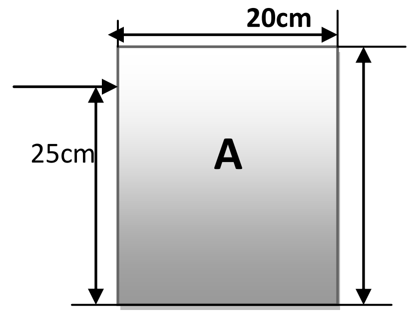
=0.4*196=78.4

F‹F^{-}

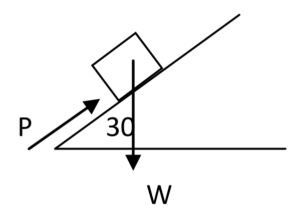
The body is equilibrium

Problems

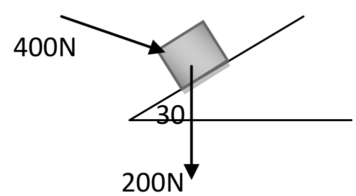
Q1:- From figure below. The solid homogeneous (400N). block(A) rests on horizontal plane. the coefficient of friction between the block and the plane is (0.34).Determine the force(P).applied as shown. Which will cause motion of (A) to impend?



Q2:- A (200N) block is contact with an inclined plane at (30^0) with the horizontal. a force (P) parallel to the inclined acting up word the plane is applied to the body if the coefficient of friction is (0.2). find the value of (P) to just case motion to impend up plane.



Q3:- in fig as shown. determine the magnitude coefficient of friction(μ)



Center of Gravity & Centriod of areas

Centroid: The centroids of an area may be defined as the point at which the area is considered to be concentrated

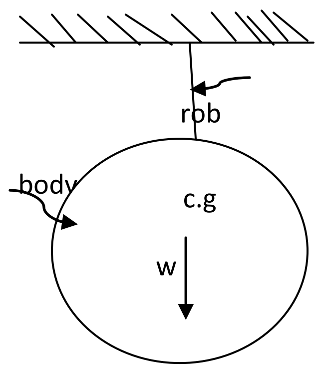
Centraidal axis: It's the axis passing through the center of gravity of a body or through the centroid of an area. The centroids for common a geometric shapes can be found by integration

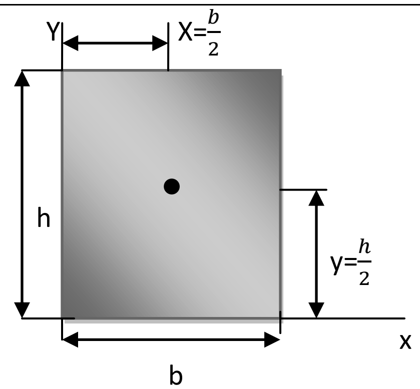
($\bar{x} = \int x dA/A$, $\bar{y} = \int y dA/A$).

From figure below. A body of weight (W) is supported by a string attached at(A). the only external forces acting on the body are its weight and the reaction exerted by the string.

Equilibrium of the body can exist only. If these two forces are equal, opposite and collinear, the line of action of the support. If we supported the body at point(B). the body will shaft its position so that the line of action of the weight is

again collinear with the string.

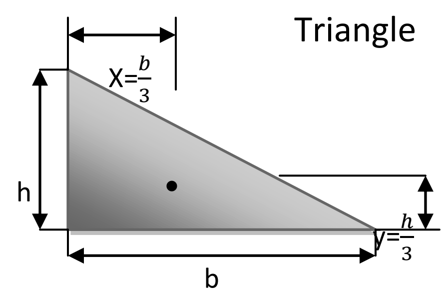




A=b*h

Centeroid=($\frac{b}{2}$,$\frac{h}{2}$)

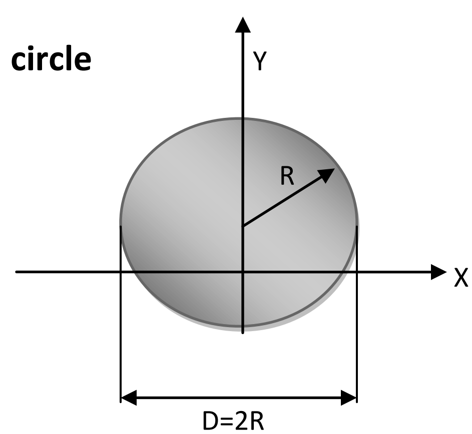
Triangle



A=b*h/2

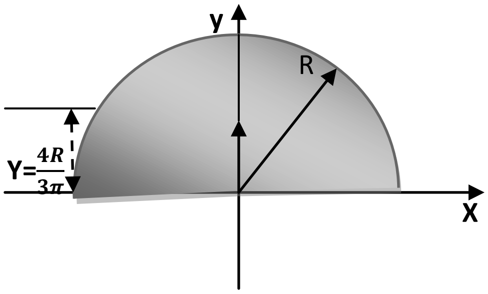
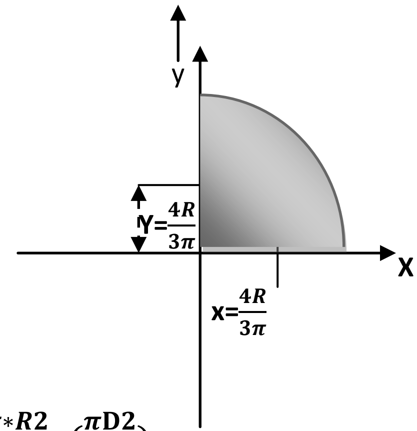
Centeroid=($\frac{b}{3}$,$\frac{h}{3}$)

circle

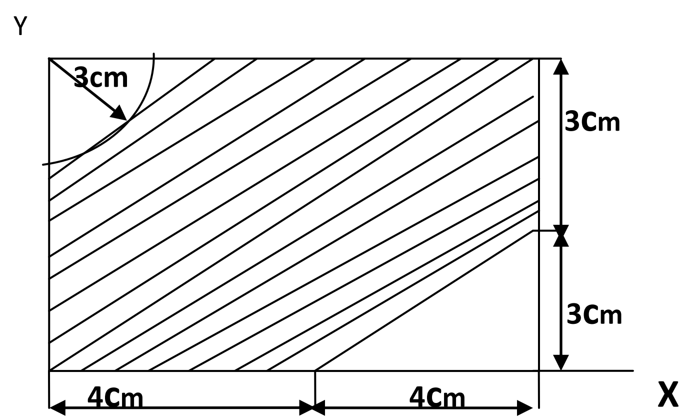


A=π*R^2=($\frac{\pi D2}{4}$)

Centeroid=(0,0)

cemi circle	quarter circle	
 A=$\frac{\pi * R2}{2}$=($\frac{\pi D2}{8}$) **Centeroid=(0, $\frac{4R}{3\pi}$)**	 A=$\frac{\pi * R2}{4}$=($\frac{\pi D2}{16}$) **Centeroid=($\frac{4R}{3\pi}$, $\frac{4R}{3\pi}$)**	

<u>**Example:-**</u> Compute the centroid for the shaded area shown in the following figure with respect to the (x- y axis).

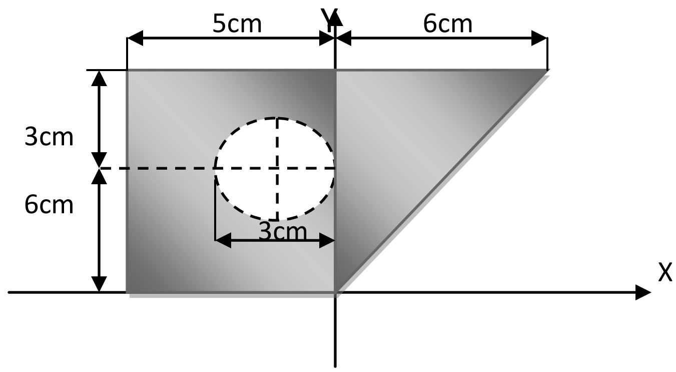


Solution:-

shape	Area(cm^2)	X(cm)	a.x	Y(cm)	a.y
Semi circle	$(r)^2$ / 2=14.14	r = 3	42.42	0.424 r + 8 = 0.424 (3) = 9.27	131
triangle	b d = 6*8 = 48	1/2 b = 6/2 = 3	144	1/2 d = 8/2 = 4	192
total	62.14		186.42		323

x- = $\sum a_i x_i$/A = 186.42/62.14 = 3 cm y- = $\sum a_i y_i$ /A= 323/62.14 = 5.2 cm

Example:- Locate the centroid for the shaded area shown in the following figure with respect to the (x- y axis).



Solution:-

A total= A1+A2-A3

A1= $\frac{6*9}{2}$**= 27 cm²**

A2= 5*9= 45 cm²

A3=$(3)^2\pi$**= 7.068 cm²**

A total= 27+45-7.068= 64.932 cm²

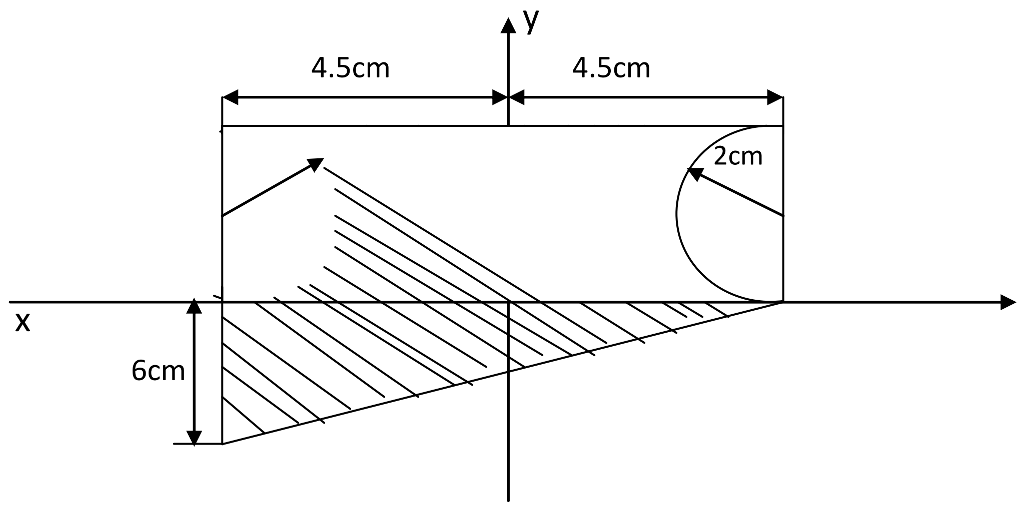
Mx= 27(6) +45(4.5)- 7.068 (6) =322.092

My= 27(2) +45(-2.5)- 7.068 (-1.5) =-47.898

$\bar{X}=\frac{My}{A}=\frac{-47.898}{64.932}=-0.737$

$\bar{y}=\frac{Mx}{A}=\frac{322.092}{64.932}=4.92$

Example:- Locate the centroid for the shaded area shown in the following figure with respect to the (x- y axis).



Solution:

A total= A1+A2-A3+A4

A1= 4*9= 36 cm^2

A2= $\frac{6*9}{2}$= 27 cm^2

A3=$(2)^2\pi/2$=6.283 cm^2

A4=$(2)^2\pi/2$=6.283 cm^2

A total=36+27-6.283-6.283=50.434

Mx= 27(-2) +36(2)- 6.283 (2)- 6.283 (2) =7.132

My= 27(-1.5) +36(0)- 6.283 (4.5-$\frac{4*2}{3\pi}$) - 6.283(-(4.5-$\frac{4*2}{3\pi}$))= -40.5

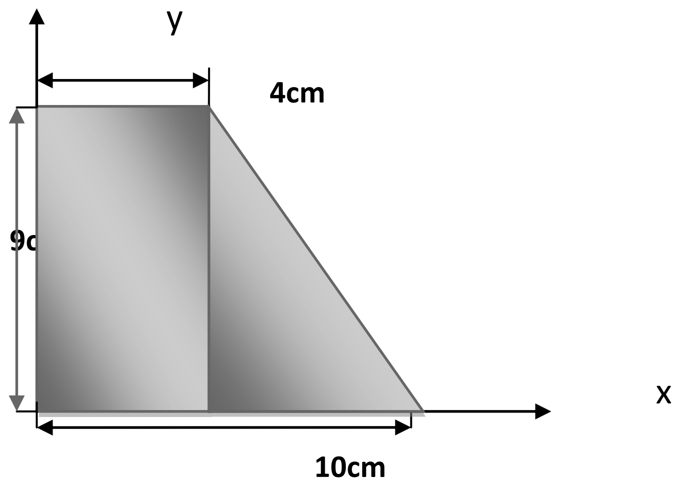
$\bar{X}=\frac{My}{A}=\frac{-40.58}{50.434}=-0.803$

$\bar{y}=\frac{Mx}{A}=\frac{-7.132}{50.434}=-0.1414$

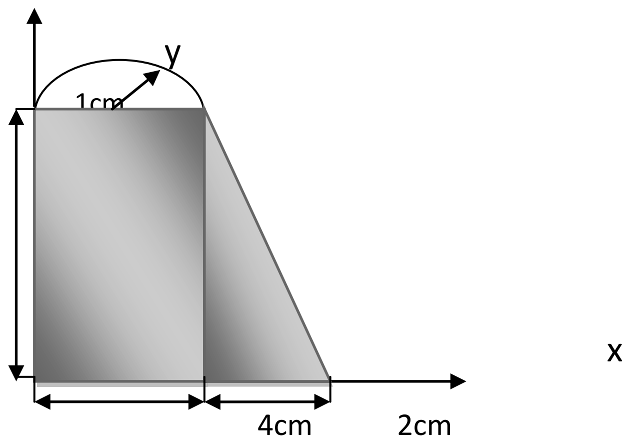
The Centeroid is-0.803, -0.1414)

Problems

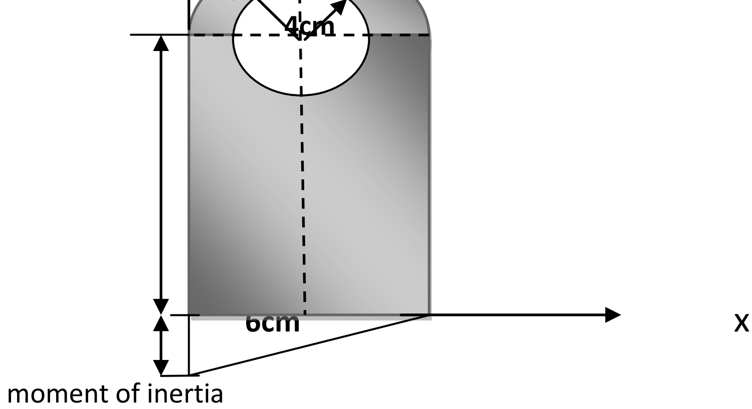
Q1:- Find the centroid for the shaded area below.



Q2:- Find the centroid for the shaded area below.



Q3:- Determine the centroid for the shaded area shown in the following figure with respect to the (x- y axis).



moment of inertia

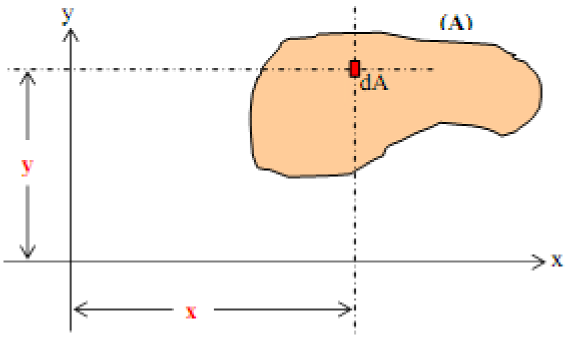
Moment of inertia: It's mathematical expression denoted by the symbol (I).

Ix = $\int y^2 dA$

Iy = $\int x^2 dA$

Where Ix and Iy: the moment of inertia about x-axis and y-axis respectively.

The units of (I) is (unit of length)^4 e,g, m^4, in^4, cm^4, ft^4...



The (I) can be transfer from the centroidal axis x_o, y_o to a parallel axis.

IX=IXo+Ad^2

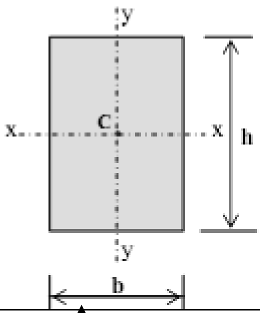
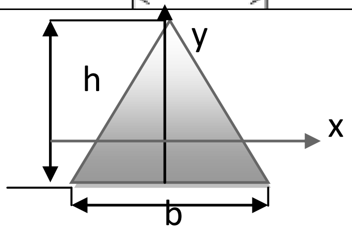
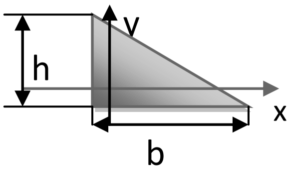
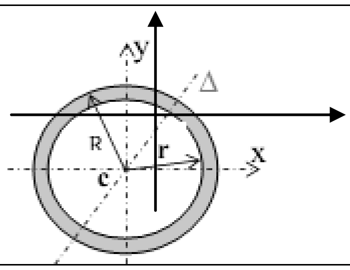
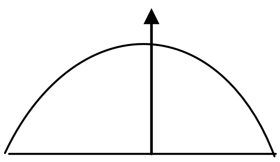
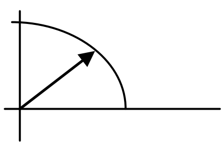
Iy=Iyo+Ad^2

$$I_x = \sum (I_{x'} + A d_y{}^2)$$

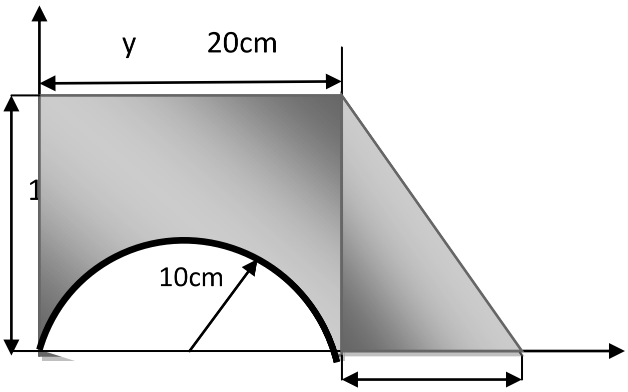
$I_{x'}$ = *Area Moment of Inertia of Each Segment*

A = *Area of Each Segment*

d_y = *Distance from Segment Centroid to Beams Centroid*

Shape	Ixo	Iyo
	$\frac{bh3}{12}$	$\frac{hb3}{12}$
	$\frac{bh3}{36}$	$\frac{hb3}{48}$
	$\frac{bh3}{36}$	$\frac{hb3}{36}$
	$\frac{\pi r4}{4}$	$\frac{\pi r4}{4}$
	$0.11r^4$	$\frac{\pi r4}{8}$
	$0.055r^4$	$0.005r^4$

Example: For the following shaded area find the moment of inertia with respect to x-axis.



,

Solution:

1- Rectangle = $I_{x1}= bh^3/12= 20(15)^3/12 = 5625\ cm^4$

$.a_1 = bd = 20(15) = 300\ cm^2$, $D^1= y_1 = d/2 = 15/2 = 7.5\ cm^4$

2-Triangle $I_1=bh^3/36 = 562.5\ cm^4$

$a_2=h/2\ (b-a) = 15/2\ (26-20) = 45\ cm^2$ or $q = 1/2\ bh$

$=6*15/2=90/2=45cm^2$

$D^2 = y_2= h/3 = 15/3 = 5\ cm$

3- Semicircle:

$I_{x3}= 6.11r^4 = 0.11\ (10)^4 = 1100\ cm^4$

$a_3= \pi\ r^2/2 = 11(10)^2/2 = 157cm^2$

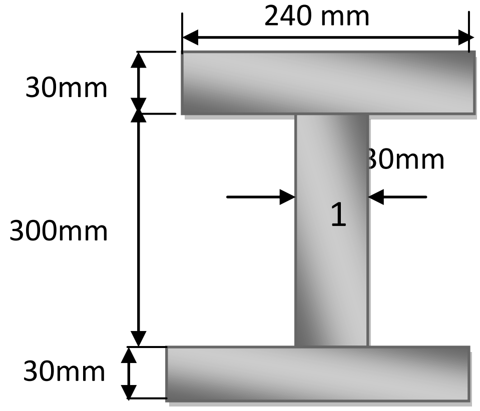
$D_3= y_3 = 0.414r = 0.424(10) = 4.24\ cm$

Shape	Ixo	area	d	d^2	Ad^2
Rectangle	5625	300	7.5	56.25	16875
Triangle	-1100	175	4.24	17.95	-2822.5
Semi circle	5625	45	5	25	1125
Total	5087.5				15177.5

$IX = Ixo + Ad^2$

$I_x= 50875 + 151775 = 20265\ cm^4$

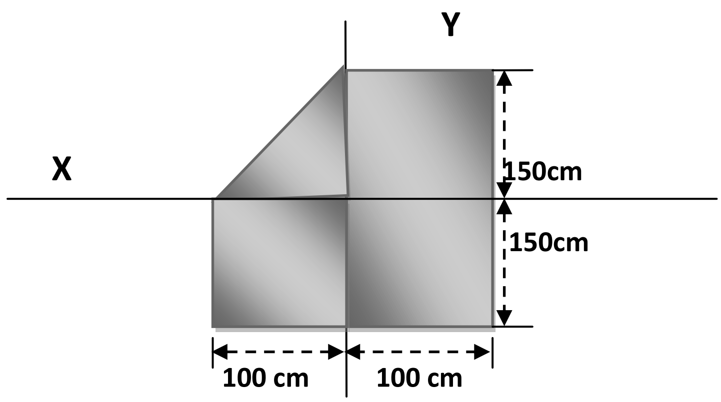
Example: **For the following shaded area find the moment of inertia with respect to x-axis.**



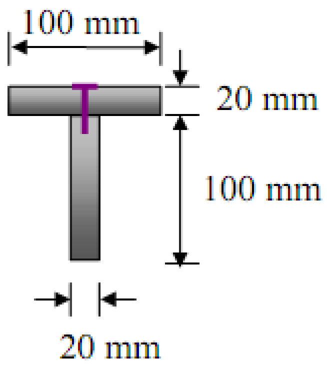
Shape	Ixo	area	d	d^2	Ad^2
Rectangle(1)	$9.331*10^8$	300	7.5	56.25	16875
Rectangle(2,3)	$4.725*10^8$	175	4.24	17.95	-2822.5
Rectangle(1)	5625	45	5	25	1125
Total	5087.5				15177.5

Problems

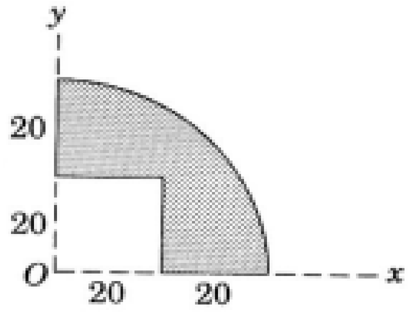
Q1:- For the following shaded area find the moment of inertia with respect to x-axis.



Q2:- For the following shaded area find the moment of inertia with respect to x-axis.



Q3:- For the following shaded area find the moment of inertia with respect to y-



Dimensions in millimeters axis.

Strength of Materials

Definitions:-

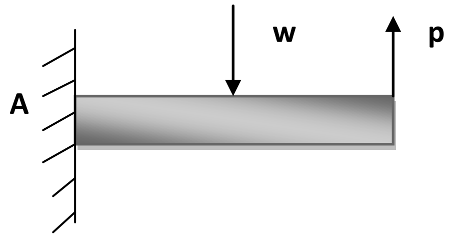
Strength of materials: is the science which deals with relations between externally applied loads and their internal effects on bodies. The bodies are not assumed to be ideal rigid as in the statics. Rigid body does not deform under any loads.

In the following figure

Static solution

The bar assumed to be rigid.

P can be calculate by ∑MA= 0

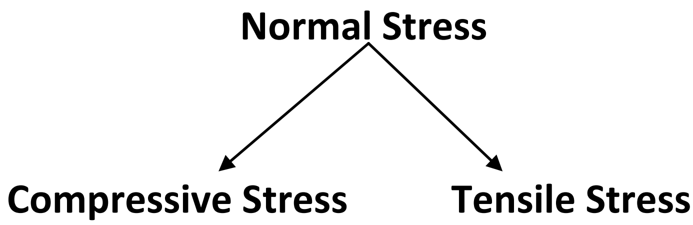


<u>In strength of materials</u>

The solution extend to study the effect of forces on the bar. We must sure that the bar will neither break nor be so flexible that it bends without lifting the load

<u>The purpose of studying strength of materials</u> is to be ensure that the structures used will be safe against the maximum internal effects that may be produced by any combination of loading

<u>Loading :.</u>Is the external force acting on a body<u>:</u>



Strain(Є):- Is the deformation accompany loading. Or is a dimensionless value, it is the ratio between the change of length to the original length:

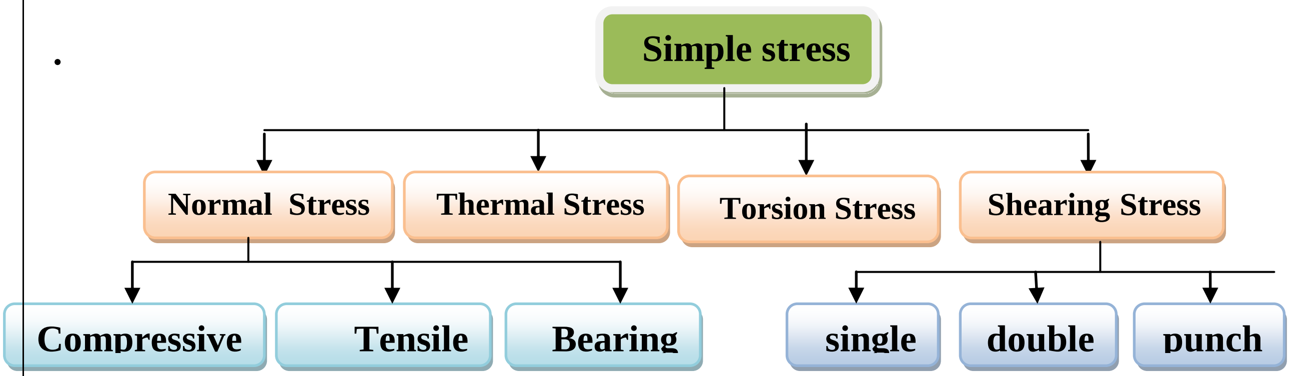
$$\epsilon = \frac{\Delta l}{LO}$$

Where:

Δl = Change in length,

LO= original length

Stress: It is the external force per unit cross-sectional area. When it is acts perpendicular to the cross-section area called normal stress.



Normal Stress due to an axial load on uniform cross section area and variable cross section area.

Normal Stress:

$(\sigma) = P / A$

Where:

P: applied load

A: cross-sectional area (normal to p)

Stress units = Force unites / Area Units = $Kg./Cm^2$, $lb./in^2$, T/m^2.

1 Kg. = 2.205 lb. and 1 in. = 2.54 cm

then 1 $Kg./Cm^2$ = 14.223 lb / in^2 , and 1 $Kg./Cm^2$ = 10.0 T/m^2

A) Compression (compressive stress) (σc)

B) Tension stress (σt):

<u>Poisson's Ratio</u> Is the ratio between lateral strain to the longitudinal strain.

$$\mu = \frac{\textit{Lateral Strain}}{\textit{Longitudinal Strain}}$$

The value of μ for all materials varies over a range of $0.0 \leq \mu \leq 0.50$

Young's Modulus

Is the ratio between stress and strain in the elastic stage

$E=\frac{strain}{stess} = \frac{\varepsilon}{\sigma}$ **(as the same stress units, (because the strain is dimensionless).**

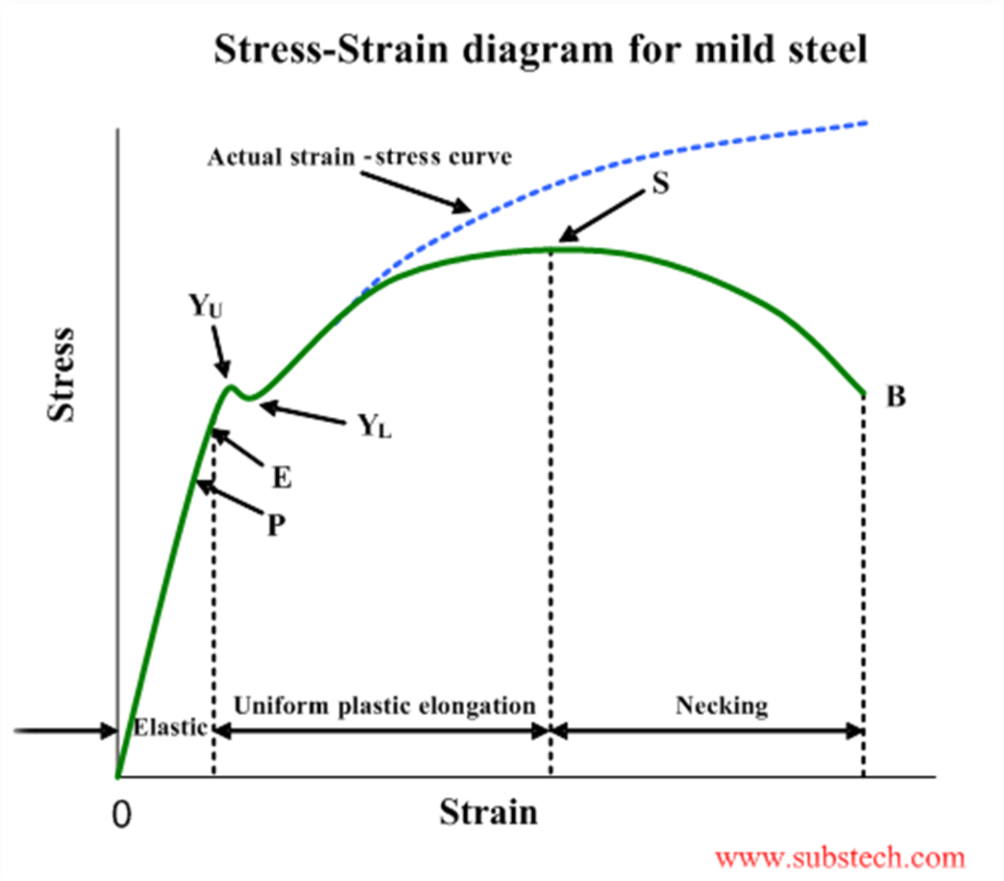
Elasticity

Is the ability of the material to return to its original dimensions when the external applied load is removed

Plasticity

Is the property which permits materials to undergo permanent change in shape without fracture, i.e the material does not return to its original dimensions

Stress-strain diagram



From figure. Represents such as a graph. Notice that we did not plot load her unit load or stress was plotted a against unit elongation. Technically known as strain only by reducing observed values to a unit basis can the properties of one specimen be compared with those of other Specimens the diagram is called a Stress-strain diagram.

The line(0E) in the Stress-Strain curve indicates the range of elastic deformation – removal of the load at any point of this part of the curve results in return of the specimen length to its original value.

The elastic behavior is characterized by the elasticity limit (stress value at the point E):

$\sigma_{el} = F_E / S_0$

For the most materials the points P and E coincide and therefore $\sigma_{el}=\sigma_p$.

A point where the stress causes sudden deformation without any increase in the force is called yield limit (yield stress, yield strength):

$\sigma_y = F_Y / S_0$

The highest stress (point Y_U) , occurring before the sudden deformation is called upper yield limit .

The lower stress value, causing the sudden deformation (point Y_L) is called lower yield limit.

The commonly used parameter of yield limit is actually lower yield limit.

If the load reaches the yield point the specimen undergoes [plastic deformation] – it does not return to its original length after removal of the load. As the load increase, the specimen continues to undergo plastic deformation and at a certain stress value its cross-section decreases due to "necking" (point S in the Stress-Strain Diagram). At this point the stress reaches the maximum value, which is called ultimate tensile strength (tensile strength):

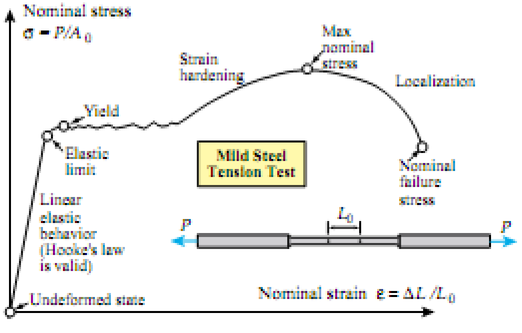
$\sigma_t = F_S / S_0$

Continuation of the deformation results in breaking the specimen - the point B in the diagram.

The actual Stress-Strain curve is obtained by taking into account the true specimen cross-section instead of the original value.

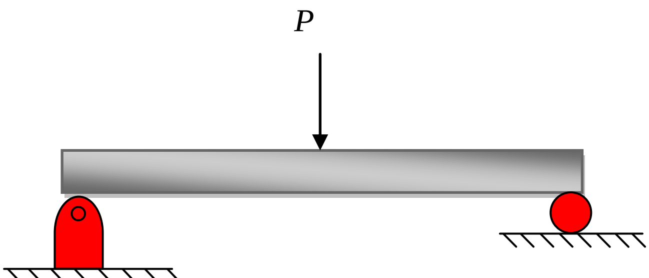
Other important characteristic of metals is ductility - ability of a material to deform under tension without rupture.

Two ductility parameters may be obtain from the tensile test:

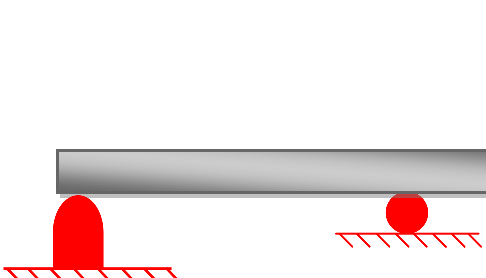


Shear and Moment Diagram:

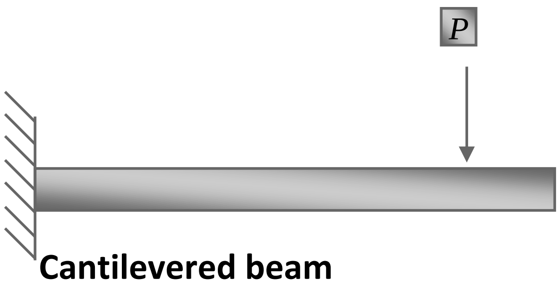
Beams are long straight members that carry loads perpendicular to their longitudinal axis. They are classified according to the way they are supported, e.g. simply supported, cantilevered, or overhanging.



Simply supported beam



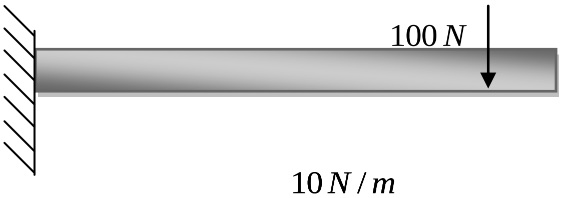
overhanging beam



Cantilevered beam

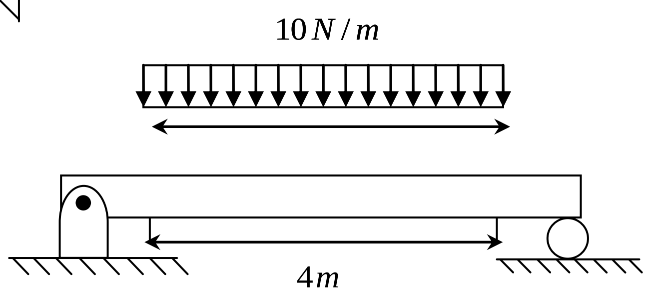
Types of Loading:

Loads commonly applied to a beam may consist of concentrated forces(applied at a point), uniformly distributed loads, in which case the magnitude is expressed as a certain number of newtons per meter of length of the beam, or uniformly varying loads. A beam may also be loaded by an applied couple.

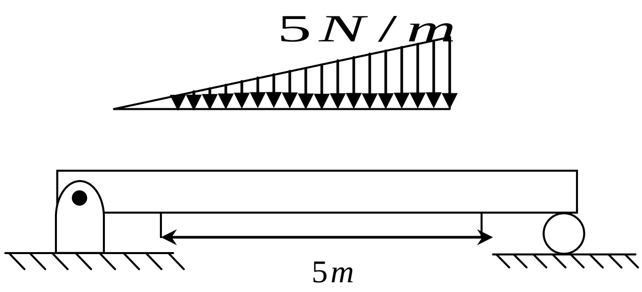


(point load)

(concentrated force)



Uniformly distributed load

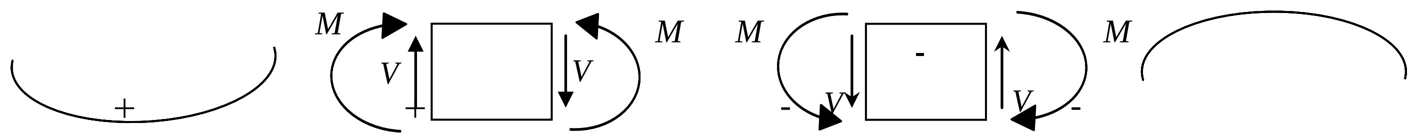


Uniformly varying load

Shearing force and bending moment diagrams show the variation of these quantities along the length of a beam for any fixed loading condition. At every section in a beam carrying transverse loads there will be resultant forces on either side of the section which, for equilibrium, must be equal and opposite.
Shearing force at the section is defined as the algebraic sum of the forces taken on one side of the section. The bending moment is defined as the algebraic sum of the moments of the forces about the section, taken on either side of the section.

Sign Convention:

Forces upwards to the left of a section or downwards to the right of a section are positive. Clockwise moments to the left and counter clockwise to the right are positive.

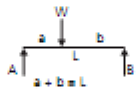
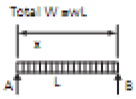
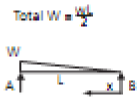
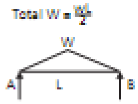
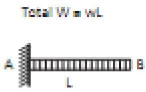
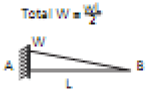
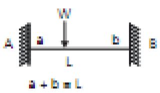
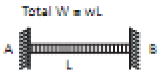
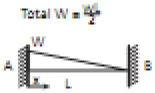
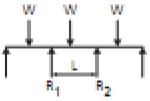
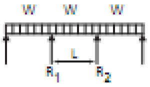


Procedure of Analysis:

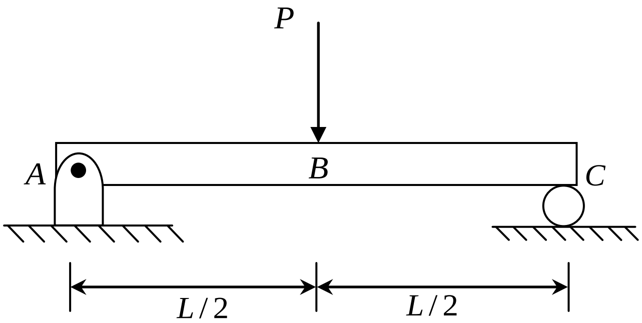
The shear and moment diagrams for a beam can be constructed using the following procedure:-

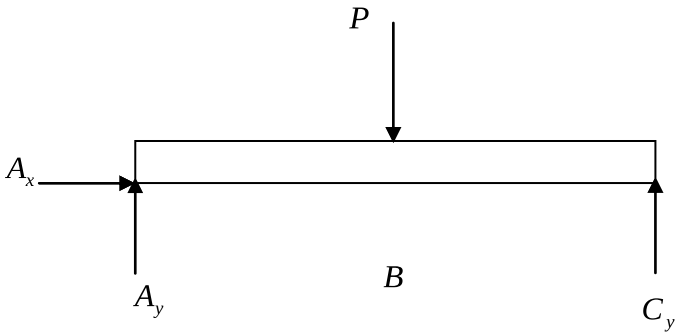
1. Determine all the reactive forces and couple moments acting on the beam, and resolve all the forces into components acting perpendicular and parallel to the beam's axis.
2. Specify separate coordinates x having an origin at the beam's left end extending to regions of the beam between concentrated forces and/or couple moments, or where there is no discontinuity of distributed loading.
3. Section the beam perpendicular to its axis at each distance x, and draw the free body diagram of one of the segments. Be sure V and M are shown acting in their positive sense, in accordance with the sign convention given as above.
4. The shear is obtained by summing forces perpendicular to the beam's axis.
5. The moment is obtained by summing moment about the sectioned end of the segment.
6. Plot the shear diagram(V versus x) and the moment diagram(M versus x). If numerical values of the functions describing V and M are positive, the values are plotted above the x-axis, whereas negative values are plotted below the axis.

Beam equations

Loading diagram	Shear force at *x*: *Qx*	Bending moment at *x*: *Mx*	Deflection at *x*: *δx*
	$Q_A = \frac{Wb}{L}$ $Q_B = -\frac{Wa}{L}$	$M_c = \frac{Wab}{L}$ When $a = b$ $M_c = \frac{WL}{4}$	$\delta_c = \frac{Wa^2b^2}{3EIL}$
	$Q_A = \frac{W}{2}$ $Q_B = -\frac{W}{2}$	$M_{max} = \frac{WL}{8}$ at $x = \frac{L}{2}$	$\delta_{max} = \frac{5WL^3}{384EI}$ at $x = \frac{L}{2}$
	$Q_A = \frac{2W}{3} = \frac{wL}{3}$ $Q_B = -\frac{W}{3} = -\frac{wL}{6}$	$M_{max} = 0.064wL^2$ at $x = 0.577L$	$\delta_{max} = 0.00652\frac{wL^4}{EI}$ at $x = 0.519$
	$Q_A = \frac{W}{2} = \frac{wL}{4}$ $Q_B = -\frac{W}{2} = -\frac{wL}{4}$	$M_{max} = \frac{wL^2}{12}$ at $x = \frac{L}{2}$	$\delta_{max} = \frac{wL^4}{120EI}$ at $x = \frac{L}{2}$
	$Q_A = Q_B = W$	$M_A = -WL$	$\delta_B = \frac{WL^3}{3EI}$
	$Q_A = W$ $Q_B = 0$	$M_A = -\frac{WL}{2} = -\frac{wL^2}{2}$	$\delta_B = \frac{WL^3}{3EI} = \frac{wL^4}{3EI}$
	$Q_A = W$ $Q_B = 0$	$M_A = -\frac{WL}{3} = -\frac{wL^2}{6}$	$\delta_B = \frac{wL^4}{30EI}$
	$Q_A = \frac{Wb}{L}$ $Q_B = -\frac{Wa}{L}$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wa^2b}{L^2}$	$\delta_C = \frac{Wa^3b^3}{3EIL}$
	$Q_A = \frac{W}{2}$ $Q_B = -\frac{W}{2}$	$M_A = M_B = -\frac{WL}{12}$	$\delta_C = \frac{WL^3}{384EI}$
	$Q_A = \frac{2W}{3}$ $Q_B = -\frac{W}{3}$	$M_A = -\frac{WL}{10} = -\frac{wL^2}{20}$ $M_B = -\frac{WL}{15} = -\frac{wL^2}{30}$	$\delta_{max} = \frac{wL^4}{764EI}$ at $x = 0.475L$
	$Q_A = \frac{W}{2}$	$M_{max} = \frac{WL}{6}$	$\delta_{max} = \frac{wL^3}{192EI}$
	$Q_A = \frac{W}{2}$	$M_{max} = \frac{WL}{12}$	$\delta_{max} = \frac{wL^3}{384EI}$

Example: **Draw the shear and moment diagrams for the beam shown below.**





$$\sum F_x = 0$$

$\mathbf{A_x=0}$

$$\sum M_C = 0$$

$\mathbf{P \times L/2 - A_y \times L = 0}$

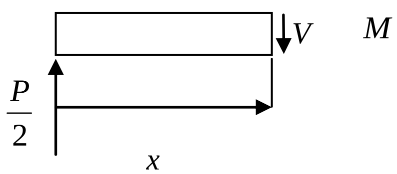
$\mathbf{A_y = P/2}$

$$\sum F_y = 0$$

$\mathbf{C_y + A_y - P = 0}$

$\mathbf{C_y = P/2}$

- **Segment AB**



$$\sum F_y = 0$$

$\frac{P}{2}$ **-V=0**

V= $\frac{P}{2}$

$$\sum M = 0$$

M- $\frac{P}{2}$ **x=0**

M= $\frac{P}{2}$ **x**

- **Segment BC**

$\sum F_y = 0$

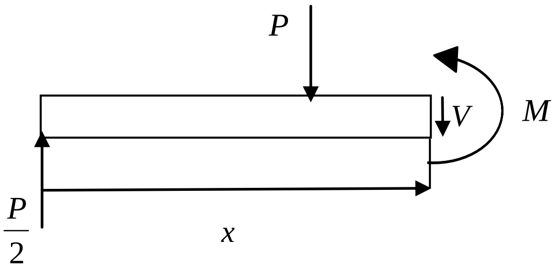
$\frac{P}{2}$ **-P-V=0**

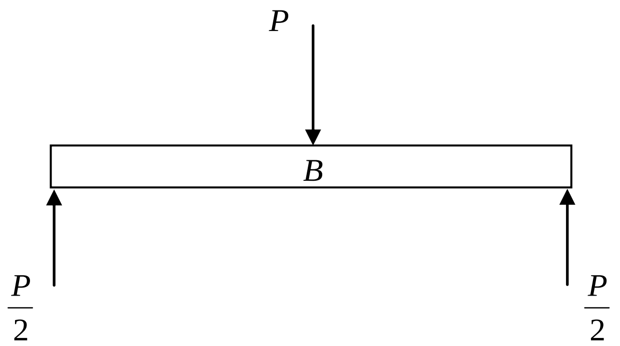
V=- $\frac{P}{2}$

$\sum M = 0$

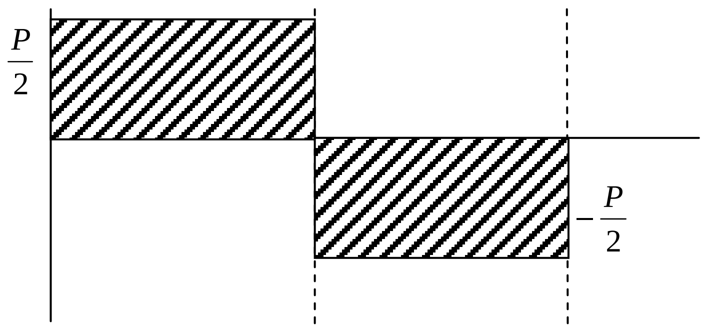
M- $\frac{P}{2}$ **×x+P(x-** $\frac{L}{2}$ **)=0**

M= $\frac{P}{2}$ **(L-x)**

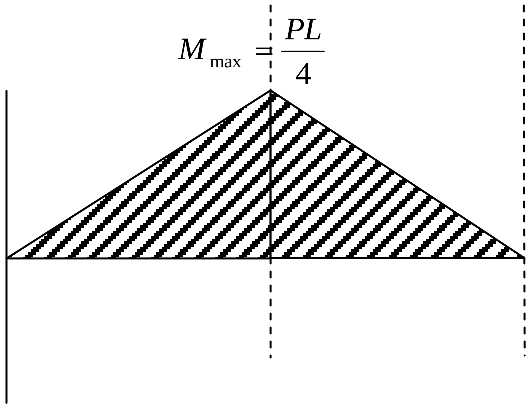




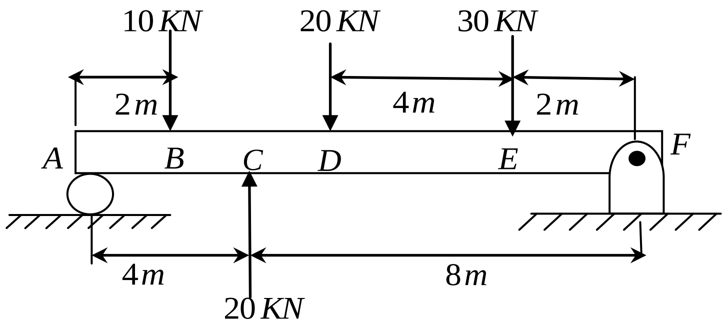
S.F. diagram



B.M. diagram



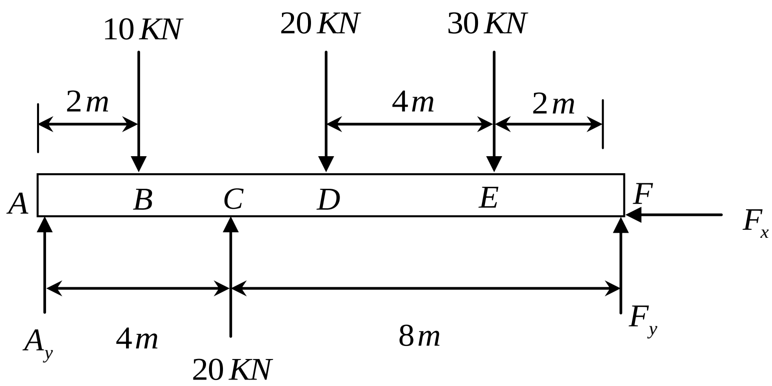
Example: **Draw the shear and moment diagrams for the beam shown below**



$\sum F_x = 0$

F_x=0

$\sum M_F = 0$



-A_y×12+10×10-20×8+20×6

+30×2=0

A_y=10 KN

$\sum F_y = 0$

10-10+20-20-30+F_y=0

F_y=30 KN

- **Segment AB** $0 \le x \le 2$

$\sum F_y = 0$

10-V=0

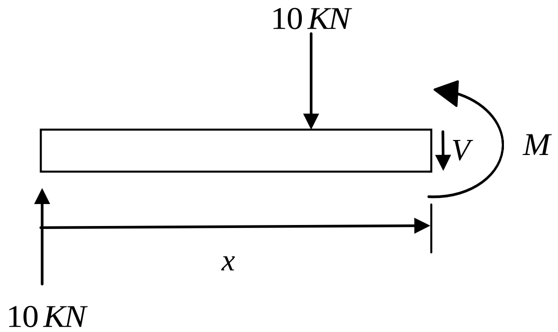
V=10 KN

$\sum M = 0$

M-10×x=0

M=10x

Segment BC $2 \le x \le 4$



$\sum F_y = 0$

10-10-V=0

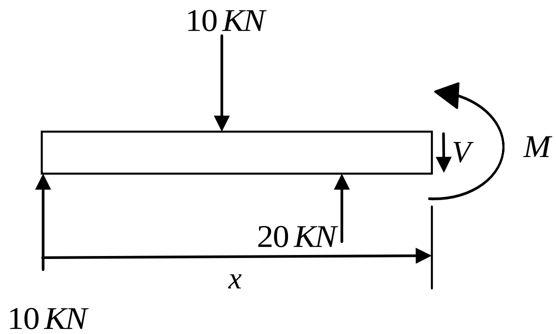
V=0

$\sum M = 0$

M-10x+10(x-2)=0

M=20 KN.m

- **Segment CD** $4 \le x \le 6$



$\sum F_y = 0$

10-10+20-V=0

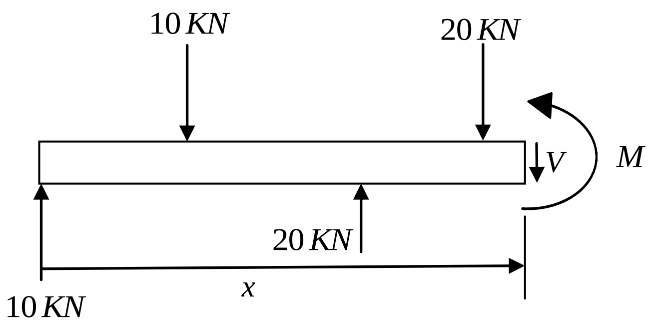
V=20 KN

$\sum M = 0$

M-10x+10(x-2)-20(x-4)=0

M=20(x-3)

- **Segment DE** $6 \le x \le 10$



$\sum F_y = 0$

10-10+20-20-V=0

V=0

$\sum M = 0$

M-10x+10(x-2)-20(x-4)+20(x-6)=0

M=60 KN.m

- **Segment EF** $10 \le x \le 12$

$\sum F_y = 0$

10-10+20-20-30-V=0

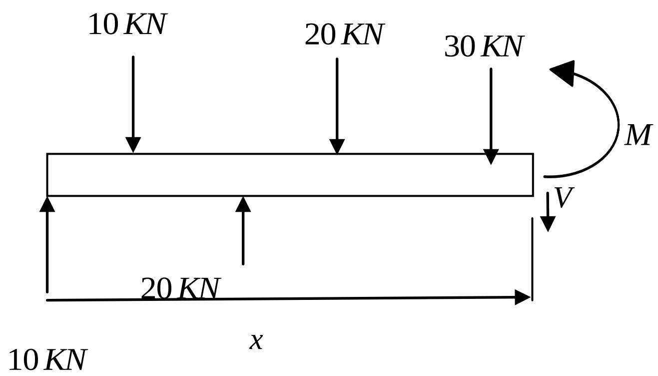
V=-30 KN

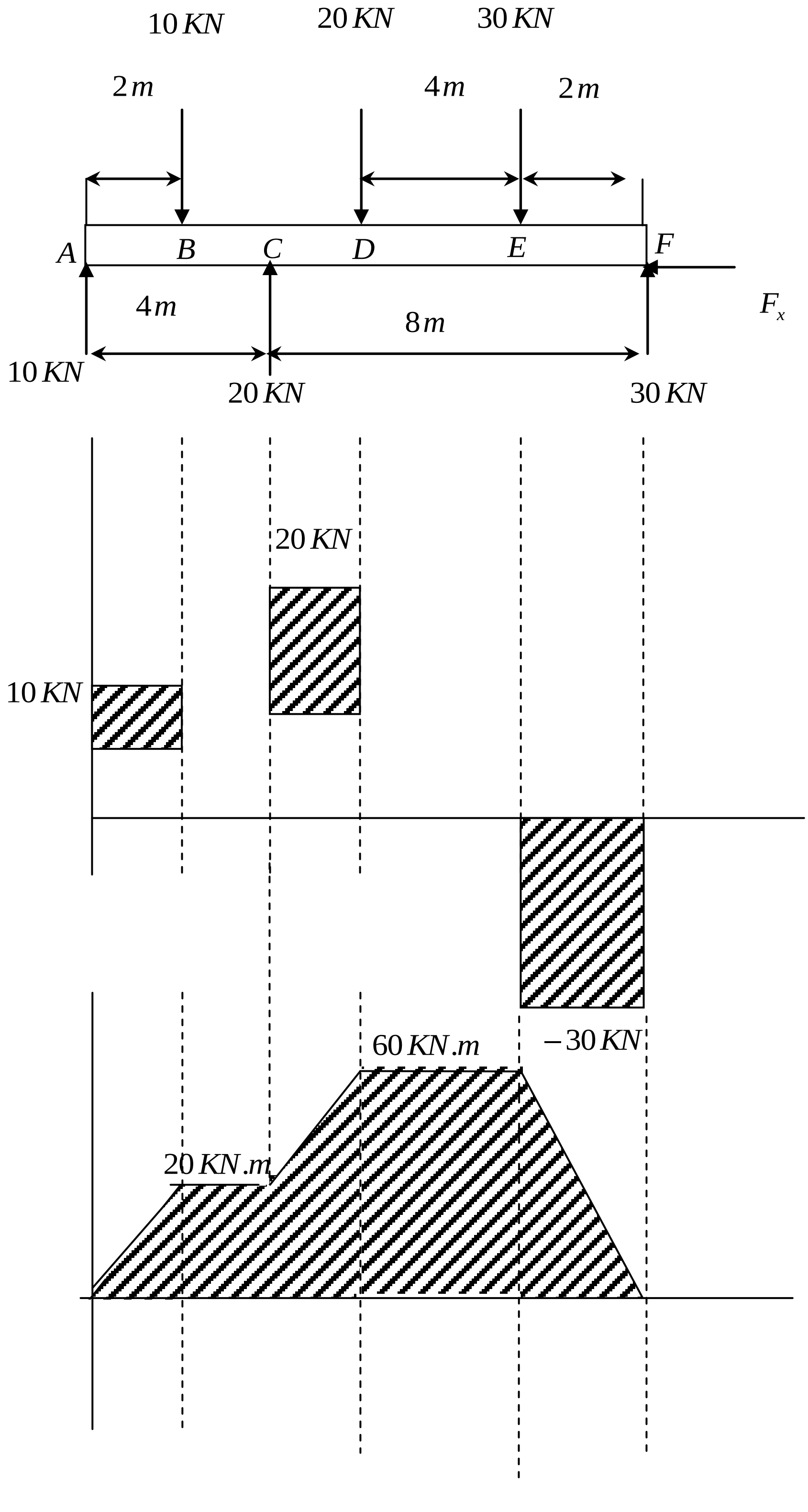
$\sum M = 0$

M-10x+10(x-2)-20(x-4)+20(x-6)

+30(x-10)=0

M=30(12-x)

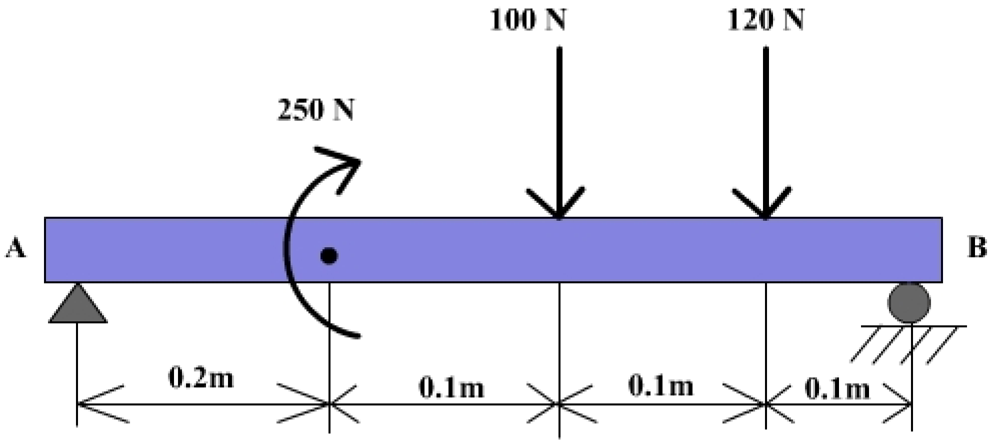




S.F Diagram

B.M. Diagram

Example: **Find the reactions at the supports for a simple beam as shown in the diagram. Weight of the beam is negligible.**



$\Sigma F_X = 0$

$\Rightarrow R_{AX} = 0$

In Y Direction

$\Sigma FY = 0$

$\Rightarrow R_{AY}+R_{BY} - 100 - 160 = 0$

$\Rightarrow R_{AY}+R_{BY} = 260$

Moment about Z axis (Taking moment about axis pasing through A)
$\Sigma M_Z = 0$

We get,

$\Sigma M_A = 0$

$\Rightarrow 0 + 250 \text{ N.m} + 100*0.3 \text{ N.m} + 120*0.4 \text{ N.m} - R_{BY} *0.5 \text{ N.m} = 0 \rightarrow$

RBY= 656N(UPWARD).

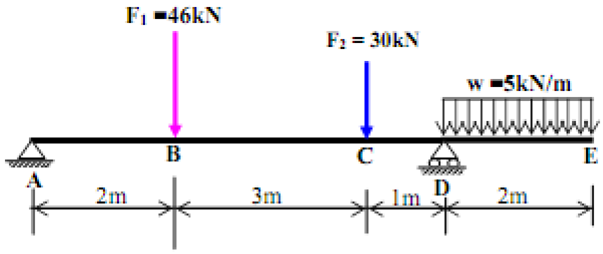
Substituting in Eq5.1 we get

$\Sigma M_B = 0$

$\Rightarrow R_{AY} * 0.5 + 250 - 100 * 0.2 - 120 * 0.1 = 0$

$\Rightarrow R_{AY} = -436$ (downwards)

Example: **Draw the shear and moment diagrams for the beam shown below**



$$\Sigma F_x = 0 \longrightarrow A_x = 0$$

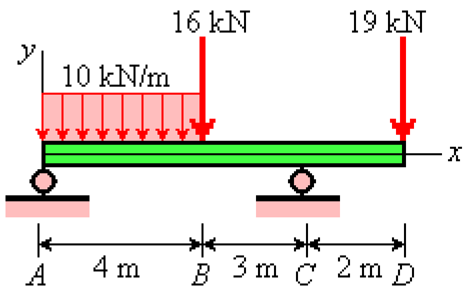
$$\Sigma M_A = 2F_1 + 5F_2 - 6D_y + 2.w.7 = 0$$
$$6D_y = 2\times 46 + 5\times 30 + 5\times 2\times 7$$
$$6D_y = 312$$
$$\longrightarrow D_y = 52 \text{ kN}$$

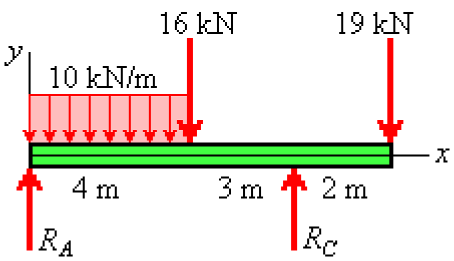
$$\Sigma F_y = A_y - F_1 - F_2 + D_y - 2.w = 0$$
$$A_y = F_1 + F_2 + 2.w - D_y$$
$$\longrightarrow A_y = 34 \text{ kN}$$

$$M_B = A_y.2 = 68 \text{ kN}$$

$$M_C = A_y.5 - F_1.3 = 32\text{kN}$$
$$M_D = -w.2.1 = -10\text{kN}$$

A beam is loaded and supported as shown in Fig. 1. For this beam





$$\circlearrowleft \sum M_A = 7R_C - 2\,[\,4 \times 10\,] - 4(16) - 9(19) = 0 \qquad (1a)$$

gives

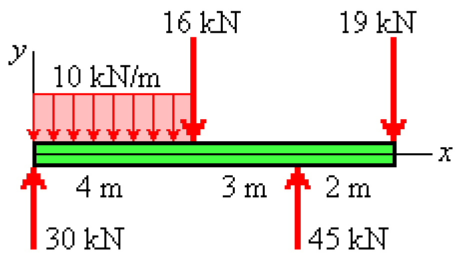
$$R_C = 45 \text{ kN} \qquad (1b)$$

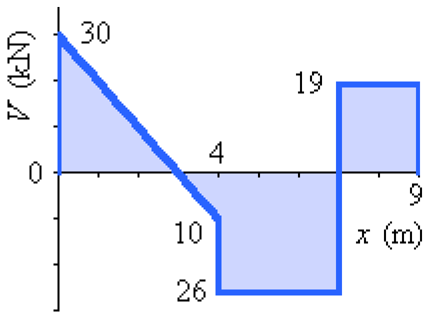
Then, summing forces in the vertical direction

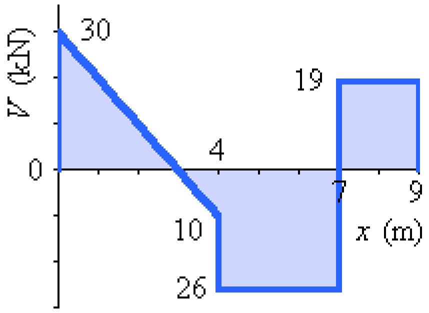
$$\uparrow \sum F = R_A + R_C - 4 \times 10 - 16 - 19 = 0 \qquad (2a)$$

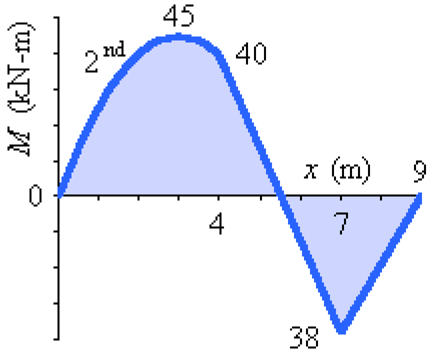
gives

$$R_A = 30 \text{ kN} \qquad (2b)$$



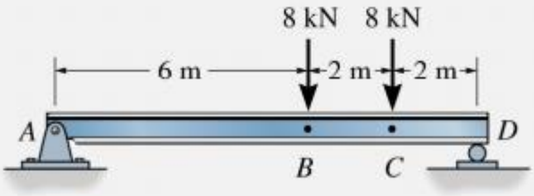



V (kN)
30
19
4
0
7
9
10
x (m)
26

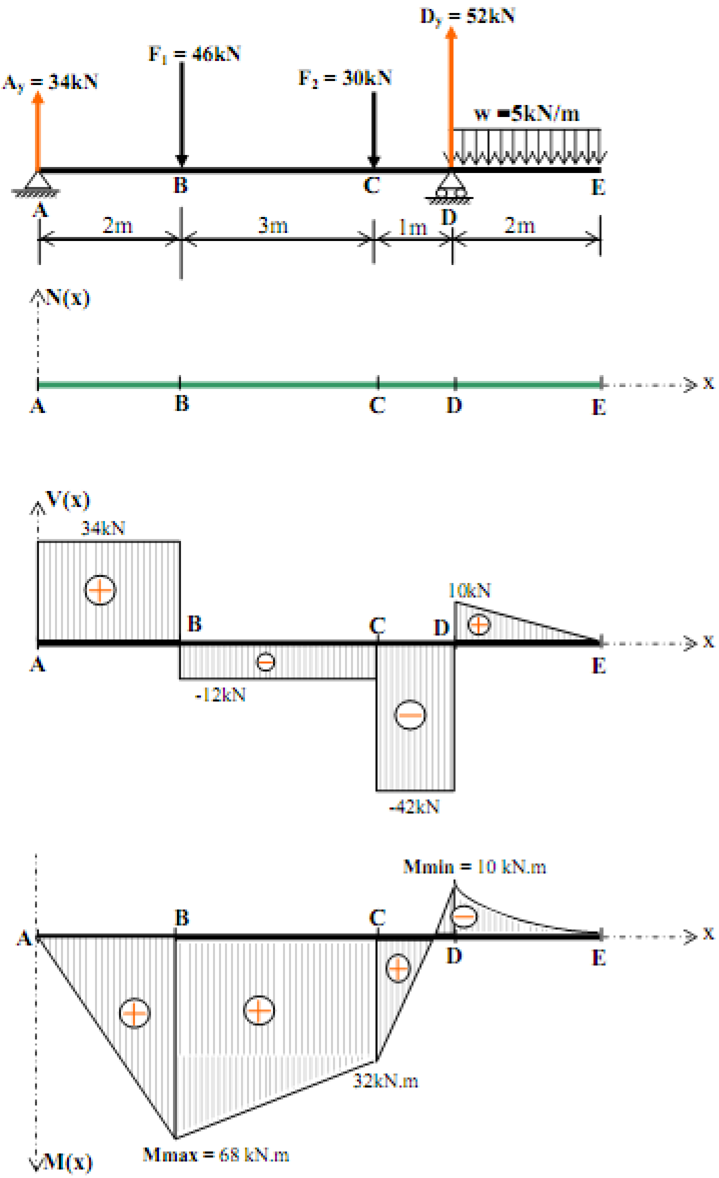

M (kN-m)
45
2nd
40
x (m)
9
0
4
7
38

EXAMPLE 6–12

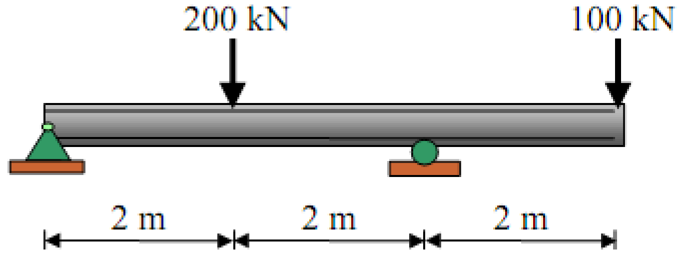
Draw the shear and moment diagrams for the beam shown in Fig. 6–18*a*.

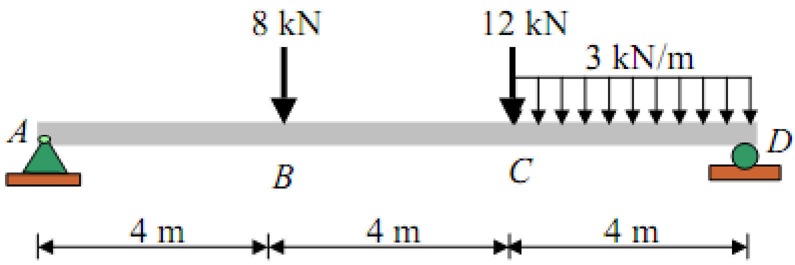


(a)



Find reaction at (A,B) and Draw the shear force diagrams for the beam.


200 kN
100 kN
2 m
2 m
2 m


8 kN
12 kN
3 kN/m
A
B
C
D
4 m
4 m
4 m

Belts

The belts are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors :

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used. It may be noted that:-

(*a*) The shafts should be properly in line to insure uniform tension across the belt section.

(*b*) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.

(*c*) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings.

(*d*) A long belt tends to swing from side to side, causing the belt to run out of the pulleys, which in turn develops crooked spots in the belt.

(*e*) The tight side of the belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.

(*f*) In order to obtain good results with flat belts, the maximum distance between the shafts should not exceed 10 metres and the minimum should not be less than 3.5 times the diameter of the larger pulley.

Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,
2. Speed reduction ratio,
3. Power to be transmitted,
4. Centre distance between the shafts,
5. Positive drive requirements,
6. Shafts layout,
7. Space available, and
8. Service conditions.

Types of Belt Drives

The belt drives are usually classified into the following three groups:

1. ***Light drives.*** These are used to transmit small powers at belt speeds up to about 10 m/s as in agricultural machines and small machine tools.

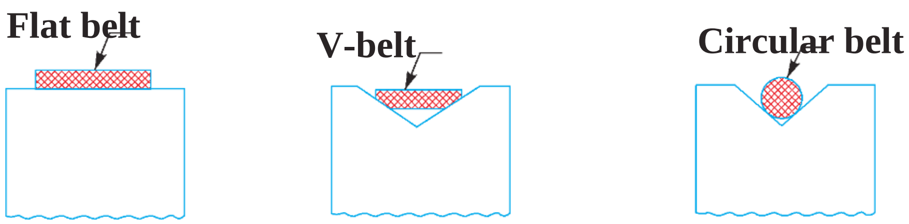
2. ***Medium drives.*** These are used to transmit medium powers at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

3. ***Heavy drives.*** These are used to transmit large powers at belt speeds above 22 m/s as in compressors and generators.

Types of Belts

Though there are many types of belts used these days, yet the following are important from the subject point of view:

1. ***Flat belt.*** The flat as shown in Fig. 18.1 (*a*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 meters apart.



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

2. ***V- belt.*** The V-belt as shown in Fig. (*b*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3. ***Circular belt or rope.*** The circular belt or rope as shown in Fig. (*c*) is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.

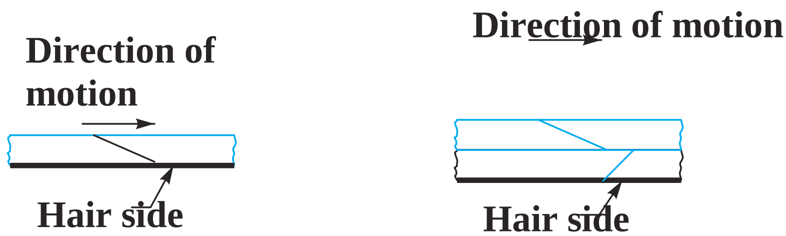
If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another

18.5 Material used for Belts

The material used for belts and ropes must be strong, flexible, and durable. It must have a high coefficient of friction. The belts, according to the material used, are classified as follows:

1. ***Leather belts.*** The most important material for flat belt is leather. The best leather belts are made from 1.2 meters to 1.5 meters long strips cut from either side of the back bone of the top grade steer hides. The hair side of the leather is smoother and harder than the flesh side, but the flesh side is stronger. The fibers on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface. Therefore for these reasons the hair side of a belt should be in contact with the pulley surface as shown in Fig. This gives a more intimate contact between belt and pulley and places the greatest tensile strength of the belt section on the outside, where the tension is maximum as the belt passes over the pulley.

The leather may be either oak-tanned or mineral salt-tanned *e.g.* chrome-tanned. In order to increase the thickness of belt, the strips are cemented together. The belts are specified according to the number of layers *e.g.* single, double or triple ply and according to the thickness of hides used *e.g.* light, medium or heavy.



(*a*) Single layer belt.   (*b*) Double layer belt.

The leather belts must be periodically cleaned and dressed or treated with a compound or dressing containing neats foot or other suitable oils so that the belt will remain soft and flexible.

2. ***Cotton or fabric belts.*** Most of the fabric belts are made by folding canvass or cotton duck to three or more layers (depending upon the thickness desired) and stitching together. These belts are woven also into a strip of the desired width and thickness. They are impregnated with some filler like linseed oil in order to make the belt water-proof and to prevent injury to the fibers. The cotton belts are cheaper and suitable in warm climates, in damp atmospheres and in exposed positions. Since the cotton belts require little attention, therefore these belts are mostly used in

farm machinery, belt conveyor etc.

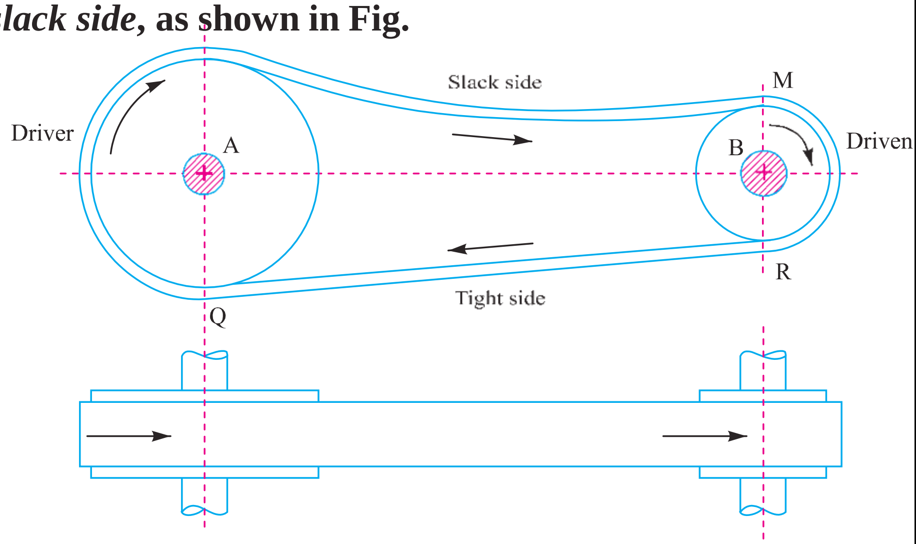
3. *Rubber belt.* The rubber belts are made of layers of fabric impregnated with rubber composition and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease. One of the principle advantage of these belts is that they may be easily made endless. These belts are found suitable for saw mills, paper mills where they are exposed to moisture.

4. *Balata belts.* These belts are similar to rubber belts except that balata gum is used in place of rubber. These belts are acid proof and water proof and it is not effected by animal oils or alkalis. The balata belts should not be at temperatures above 40°C because at this temperature the balata begins to soften and becomes sticky. The strength of balata belts is 25 per cent higher than rubber belts.

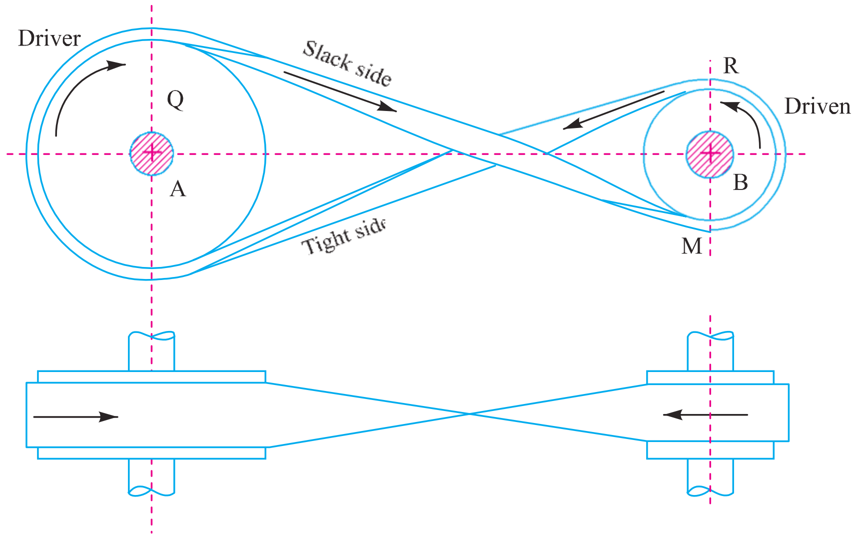
Types of Flat Belt Drives

The power from one pulley to another may be transmitted by any of the following types of belt drives.

1. *Open belt drive.* The open belt drive, as shown in Fig., is used with shafts arranged parallel and rotating in the same direction. In this case, the driver *A* pulls the belt from one side (*i.e.* lower side *RQ*) and delivers it to the other side (*i.e.* upper side *LM*). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as *tight side* whereas the upper side belt (because of less tension) is known as *slack side*, as shown in Fig.



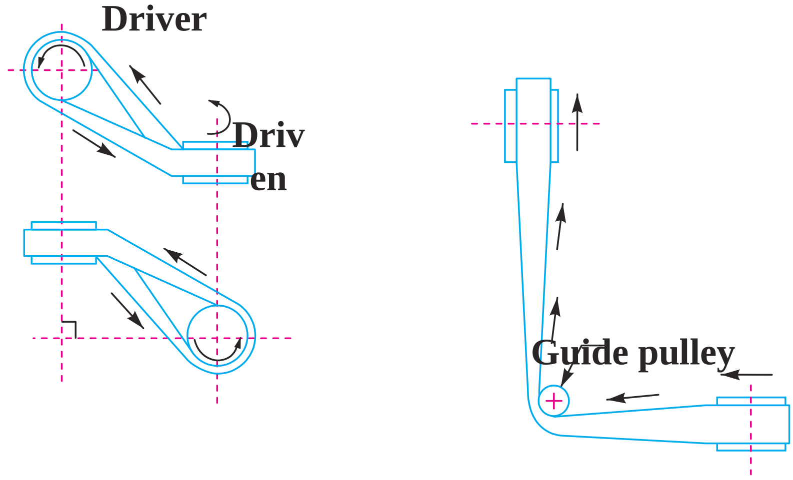
2. *Crossed or twist belt drive.* **The crossed or twist belt drive, as shown in Fig. 18.5, is used with shafts arranged parallel and rotating in the opposite directions. In this case, the driver pulls the belt from one side (*i.e. RQ*) and delivers it to the other side (*i.e. LM*). Thus, the tension in the belt *RQ* will be more than that in the belt *LM*. The belt *RQ* (because of more tension) is known as *tight side*, whereas the belt *LM* (because of less tension) is known as *slack side*, as shown in Fig. 18.5.**



A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of 20 *b*, where *b* is the width of belt and the speed of the belt should be less than
15 m/s.

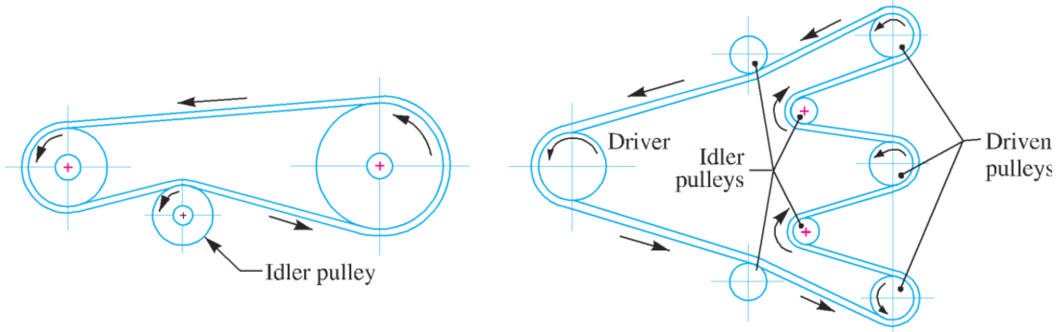
3. ***Quarter turn belt drive.*** **The quarter turn belt drive (also known as *right angle belt drive*) as shown in Fig. (*a*), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 *b*, where *b* is width of belt.**

In case the pulleys cannot be arranged as shown in Fig. (*a*) or when the reversible motion is desired, then a *quarter turn belt drive with a guide pulley*, as shown in Fig. (*b*), may be used.



(*a*) Quarter turn belt drive. (*b*) Quarter turn belt drive with guide pulley.

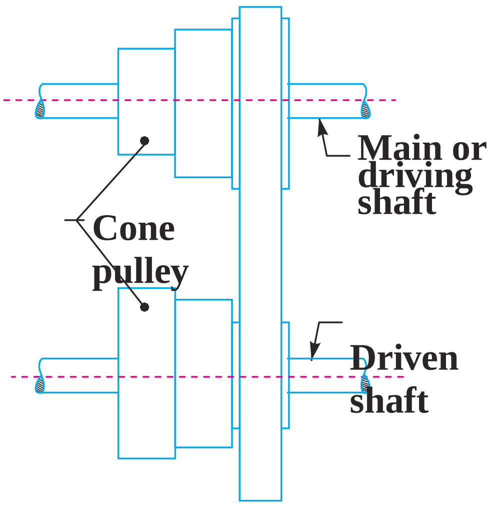
4. ***Belt drive with idler pulleys.*** **A belt drive with an idler pulley (also known as *jockey pulley drive*) as shown in Fig., is used with shafts arranged parallel and when an open belt drive can not be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension can not be obtained by other means.**

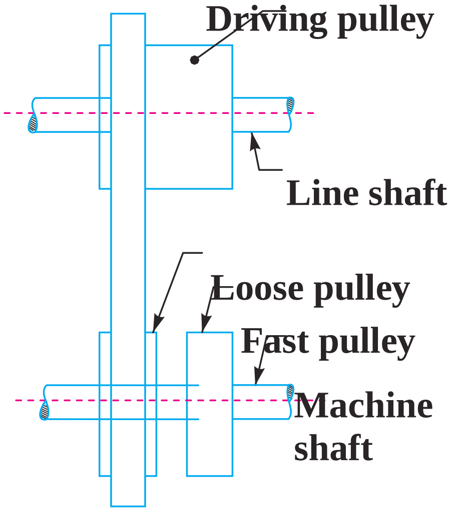


When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. 18.8, may be employed.

5. *Compound belt drive*. A compound belt drive as shown in Fig. is used when power is transmitted from one shaft to another through a number of pulleys.

6. *Stepped or cone pulley drive.* A stepped or cone pulley drive, as shown in Fig. 18.10, is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

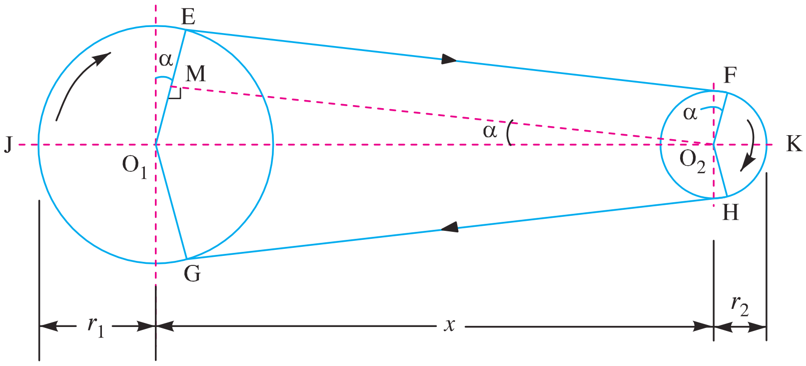




7. *Fast and loose pulley drive.* A fast and loose pulley drive, as shown in Fig. 18.11, is used when the driven or machine shaft is to be started or stopped whenever desired without interferring with the driving shaft. A pulley which is keyed to the machine shaft is called fast pulley and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.

Length of an Open Belt Drive

We have discussed in Art. that in an open belt drive, both the pulleys rotate in the same direction as shown in Fig. 18.13.



Let:- r_1 and r_2 = Radii of the larger and smaller pulleys,
x = Distance between the centers of two pulleys ($i.e.$ $O_1 O_2$), and
L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. Through O2 draw O2M parallel to FE. From the geometry of the figure, we find that O2M will be perpendicular to O1E.
Let the angle MO2O1 = α radians.
We know that the length of the belt,
L=Arc GJE + EF + Arc FKH + HG

=2 (Arc JE + EF + Arc FK)...(i)

From the geometry of the figure, we also find that

$$\sin \alpha = \frac{O1M}{o1o2} = \frac{O1E - EM}{o1o2} = \frac{r1-r2}{x}$$

Since the angle 6 is very small, therefore putting

$$\sin \alpha = \alpha \text{ (in radians)} = = \frac{r1-r2}{x} \text{...(ii)}$$

Arc JE=...(iii) 2 (r

$= \pi (r_1 + r_2) + 2\alpha (r_1 + r_2) + 2x –$ ————

$(r_1 + r_2)$

$2x$

$+ r)^2$

$(r + r)^2$

– $Arc JE=...(iii) $= \pi (r_1 + r_2) + 2x +$... (in terms of pulley radii)

$= \frac{}{2} (d_1 + d_2) + 2x + \frac{1 \quad 2}{4x}$

=... (in terms of pulley radii) =... (in terms of pulley diameters)

velocity

It is the ratio between the velocities of the driver and the follower or driven. It may beexpressed, mathematically, as discussed below:

Letd=Diameter of the driver,

d=Diameter of the follower,

N=Speed of the driver in r.p.m.,

N=Speed of the follower in r.p.m.,

$ Length of the belt that passes over the driver, in one minute

=%"d N

Similarly, length of the belt that passes over the follower, in one minute

=%"d N

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore%"d N=%"d N

%"d N=%"d N

and velocity ratio,

NdNd

When thickness of the belt (t) is considered, then velocity ratio,

Ndt

Ndt

Notes : 1. The velocity ratio of a belt drive may also be obtained as discussed below:

We know that the peripheral velocity of the belt on the driving pulley,

dN% m/s

60

%%

Ndd SpeedoflastdrivenProductofdiametersofdrivers

Ndd SpeedoffirstdriverProductofdiametersofdrivens

... (in terms of pulley diameters)

18.13 Velocity Ratio of a Belt Drive

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let d_1 = Diameter of the driver,

d_2 = Diameter of the follower,

N_1 = Speed of the driver in r.p.m.,

N_2 = Speed of the follower in r.p.m.,

$\therefore$ Length of the belt that passes over the driver, in one minute

$$= \pi d_1 N_1$$

Similarly, length of the belt that passes over the follower, in one minute

$$= \pi d_2 N_2$$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\because \quad \pi d_1 N_1 = \pi d_2 N_2$$

and velocity ratio,
$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

When thickness of the belt (t) is considered, then velocity ratio,

$$= \frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Notes : 1. The velocity ratio of a belt drive may also be obtained as discussed below:

We know that the peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi d_1 N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven pulley,

$$v_2 = \frac{\pi d_2 N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \quad \frac{\pi d_1 N_1}{\pi d_2 N_2} = \text{ or } \frac{N_2}{N_1} = \frac{d_1}{}$$

60 60 N_1 d_2

2. In case of a compound belt drive as shown in Fig. 18.7, the velocity ratio is given by

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{or} \quad \frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

.

18.17 Length of a Cross Belt Drive

We have discussed in Art. 18.12 that in a cross belt drive, both the pulleys rotate in the opposite directions as shown in Fig. 18.14.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centers of two pulleys (*i.e.* O_1O_2), and

L = Total length of the belt.

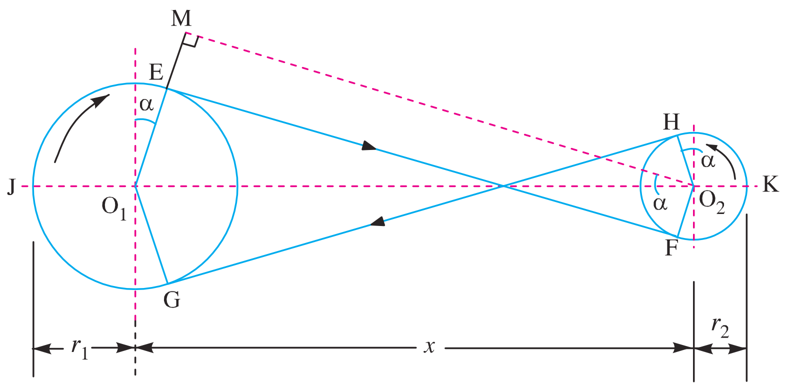
Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. 18.14.

Through O_2 draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E. Let the angle MO_2O_1 = α radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$
$$= 2\,(\text{Arc } JE + FE + \text{Arc } FK) \quad ...(i)$$



$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \quad \text{... (in terms of pulley radii)}$$

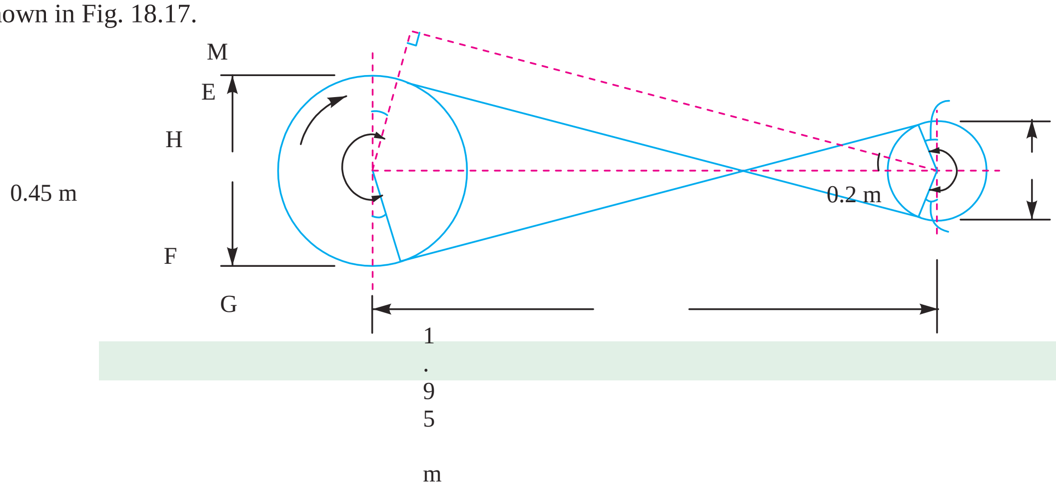
$$= \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \quad \text{... (in terms of pulley diameters)}$$

Example 18.2. *Two pulleys, one 450 mm diameter and the other 200 mm diameter, on parallel shafts 1.95 m apart are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley.*

What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25?

Solution. Given : d_1 = 450 mm = 0.45 m or r_1 = 0.225 m ; d_2 = 200 mm = 0.2 m or r_2 = 0.1 m ; x = 1.95 m ; N_1 = 200 r.p.m. ; T_1 = 1 kN = 1000 N ; μ = 0.25

The arrangement of crossed belt drive is shown in Fig. 18.17.



Length of the belt

We know that length of the belt,

$$L = \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi (0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95}$$

$$= 1.02 + 3.9 + 0.054 = 4.974 \text{ m } \textbf{Ans.}$$

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin \alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667$$

$\therefore$ $\alpha = 9.6°$

and $\theta = 180° + 2\alpha = 180 + 2 \times 9.6 = 199.2°$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad } \textbf{Ans.}$$

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right)$$

11. A flat belt, 8 mm thick and 100 mm wide transmits power between two pulleys, running at 1600 m/min. The mass of the belt is 0.9 kg/m length. The angle of lap in the smaller pulley is 165° and the coefficient of friction between the belt and pulleys is 0.3. If the maximum permissible stress in the belt is 2 MN/m^2, find (*i*) Maximum power transmitted, and (*ii*) Initial tension in the belt.

[Ans. 14.821 kW; 1.322 kN]

12. Design a flat belt drive to transmit 110 kW at a belt speed of 25 m/s between two pulleys of diameters 250 mm and 400 mm having a pulley centre distance of 1 metre. The allowable belt stress is 8.5 MPa and the belts are available having a thickness to width ratio of 0.1 and a material density of 1100 kg/m^3. Given that the coefficient of friction between the belt and pulleys is 0.3, determine the minimum required belt width.

    What would be the necessary installation force between the pulley bearings and what will be the force between the pulley bearings when the full power is transmitted?

13. A 8 mm thick leather open belt connects two flat pulleys. The smaller pulley is 300 mm diameter and runs at 200 r.p.m. The angle of lap of this pulley is 160° and the coefficient of friction between the belt and the pulley is 0.25. The belt is on the point of slipping when 3 kW is transmitted. The safe working stress in the belt material is 1.6 N/mm^2. Determine the required width of the belt for 20% overload capacity. The initial tension may be taken equal to the mean of the driving tensions. It is proposed to increase the power transmitting capacity of the drive by adopting one of the following alternatives :

    1. by increasing initial tension by 10%, and
    2. by increasing the coefficient of friction to 0.3 by applying a dressing to the belt.

    Examine the two alternatives and recommend the one which will be more effective. How much power would the drive transmit adopting either of the two alternatives?

QUESTIONS

OBJECTIVE TYPE QUESTIONS

1. The material suitable for the belts used in agricultural equipments is
   (*a*) cotton (*b*) rubber
   (*c*) leather (*d*) balata gum
2. The power transmitted by means of a belt depends upon
   (*a*) velocity of the belt
   (*b*) tension under which the belt is placed on the pulleys
   (*c*) arc of contact between the belt and the smaller pulley
   (*d*) all of the above
3. When the speed of belt increases,
   (*a*) the coefficient of friction between the belt and pulley increases
   (*b*) the coefficient of friction between the belt and pulley decreases
   (*c*) the power transmitted will decrease
   (*d*) the power transmitted will increase
4. In a crossed belt drive, the shafts are arranged parallel and rotate in the .... directions.
   (*a*) same (*b*) opposite
5. The tension in the slack side of the belt is .... the tension in the tight side of the belt.
   (*a*) equal to (*b*) less than (*c*) greater than
6. In a flat belt drive, the belt can be subjected to a maximum tension (T) and centrifugal tension (T_C). The condition for transmission of maximum power is given by
   (*a*) $T = T_C$ (*b*) $T = 2\,T_C$
   (*c*) $T = 3\,T_C$ (*d*) $T = \sqrt{3}\,T_C$
7. When a belt drive is transmitting maximum power,
   (*a*) effective tension is equal to the centrifugal tension
   (*b*) effective tension is half of the centrifugal tension
   (*c*) driving tension in slack side is equal to the centrifugal tension
   (*d*) driving tension in tight side is twice the centrifugal tension
8. All stresses produced in a belt are
   (*a*) compressive stresses (*b*) tensile stresses
   (*c*) both tensile and compressive stresses (*d*) shear stresses
9. For maximum power, the velocity of the belt will be
   (*a*) $\sqrt{\dfrac{T}{m}}$ (*b*) $\sqrt{\dfrac{T}{2m}}$ (*c*) $\sqrt{\dfrac{T}{3m}}$

10. The centrifugal tension in the belt

(a) increases the power transmitted

(b) decreases the power transmitted

(c) has no effect on the power transmitted

(d) is equal to maximum tension on the belt

ANSWERS

1. (*b*)	**2.** (*d*)	**3.** (*d*)	**4.** (*b*)	**5.** (*b*)
6. (*c*)	**7.** (*d*)	**8.** (*b*)	**9.** (*c*)	**10.** (*c*)